Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension
Insert

Friday 15 June 2018 – Afternoon
Time allowed: 2 hours

7088436939

INFORMATION FOR CANDIDATES
- This Insert contains the article for Section B.
- This document consists of **4** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR
- Do not send this Insert for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Arithmetic and Geometric Means

Arithmetic and geometric mean of two numbers

For two real numbers a and b, the arithmetic mean of the numbers is defined to be $\frac{a+b}{2}$. For two non-negative real numbers a and b, the geometric mean of the two numbers is defined to be $\sqrt{ab}$.

Squares of real numbers cannot be negative, so we know that $(a-b)^2 \geqslant 0$. It follows that $a^2+b^2 \geqslant 2ab$ and so $(a+b)^2 \geqslant 4ab$. Hence the arithmetic mean of two real non-negative numbers is greater than, or equal to, their geometric mean.

$$\frac{a+b}{2} \geqslant \sqrt{ab} \text{ for } a, b \geqslant 0$$

This result is known as the inequality of the arithmetic and geometric means. If the two numbers a and b are equal then the arithmetic mean equals the geometric mean.

The three real numbers a, $\frac{a+b}{2}$, b form an arithmetic sequence. The three non-negative real numbers a, $\sqrt{ab}$, b form a geometric sequence.

Constructing the arithmetic and geometric mean of two numbers

Lengths representing the arithmetic and geometric mean of two positive numbers can be constructed with a straight edge and compasses.

Fig. C1.1 shows a straight line ACB with AC of length a and CB of length b.

Fig. C1.1

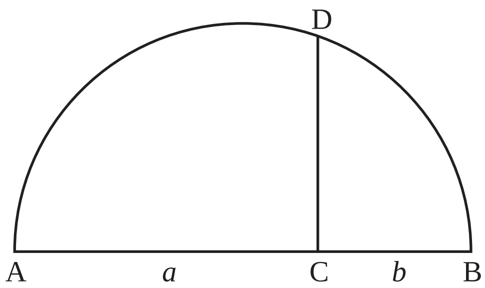


Fig. C1.2

The line AB is first bisected, to locate its midpoint. A semicircle with AB as diameter is then drawn, and a line at C perpendicular to the diameter is constructed. Fig. C1.2 shows this semicircle, with the perpendicular line through C meeting the semicircle at D.

The radius of the semicircle is the arithmetic mean of a and b, and the length of CD is the geometric mean of a and b.

To prove that the length of CD is the geometric mean of a and b, consider triangles ACD and BCD, as shown in Fig. C1.3. Letting angle CBD = θ, it follows that angle CDA is also θ. Finding expressions for $\tan\theta$ in each of triangles ACD and BCD leads to $h = \sqrt{ab}$, where h is the length of CD.

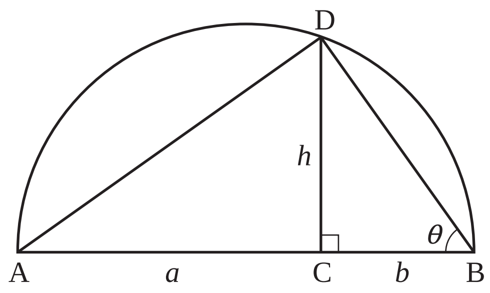


Fig. C1.3

The relationship between a, b and h in Fig. C1.3 means that a square with side CD has the same area as a rectangle with sides equal to AC and CB. Fig. C2 shows the square and a rectangle ACFG with CF equal in length to CB. This diagram illustrates how a straight edge and compasses can be used to construct a square with area equal to that of a given rectangle. This method appears in Euclid's books on Geometry (the 'Elements') which were published around 2300 years ago.

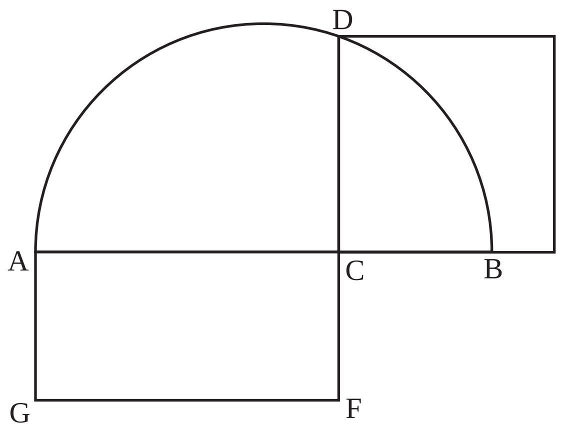


Fig. C2

Areas of rectangles

The inequality of arithmetic and geometric means implies that the square has the smallest perimeter of all rectangles with the same area.

Consider a rectangle of given area A that has sides of lengths x and y, so that $xy = A$. The perimeter of this rectangle is $2(x + y)$. From the inequality of arithmetic and geometric means, we know that $\frac{x+y}{2} \geqslant \sqrt{xy}$ so that $2(x+y) \geqslant 4\sqrt{xy}$. But the right-hand side of this last inequality has the fixed value $4\sqrt{A}$ whatever x and y are. For a square of area A, each side has length $\sqrt{A}$ and so $4\sqrt{A}$ is the perimeter of this square. Therefore, the perimeter of any rectangle of area A is not less than this, so the square has the smallest perimeter of all rectangles with given area.

Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension
Question Paper

Friday 15 June 2018 – Afternoon
Time allowed: 2 hours

7021023250

You must have:
- Printed Answer Booklet
- Insert

You may use:
- a scientific or graphical calculator

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** If additional space is required, you should use the lined page(s) at the end of the Printed Answer Booklet. The question number(s) must be clearly shown.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION
- The total mark for this paper is **75**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **8** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$$S_n = \tfrac{1}{2}n(a+l) = \tfrac{1}{2}n\{2a+(n-1)d\}$$

Geometric series

$$S_n = \frac{a(1-r^n)}{1-r}$$

$$S_\infty = \frac{a}{1-r} \text{ for } |r| < 1$$

Binomial series

$$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$$

where $^nC_r = {}_nC_r = \binom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$$\mathrm{f}'(x) = \lim_{h\to 0}\frac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$$

Integration

$$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$$

$$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,dx \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $f(x)=0$: $x_{n+1} = x_n - \dfrac{f(x_n)}{f'(x_n)}$

Probability

$P(A \cup B) = P(A)+P(B)-P(A \cap B)$

$P(A \cap B) = P(A)P(B|A) = P(B)P(A|B)$ **or** $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \sum(x_i-\bar{x})^2 = \sum x_i^2 - \dfrac{(\sum x_i)^2}{n} = \sum x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim B(n, p)$ then $P(X = r) = {}^nC_r\, p^r q^{n-r}$ where $q = 1-p$

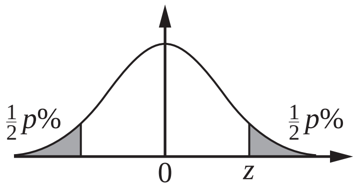
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim N(\mu, \sigma^2)$ then $\bar{X} \sim N\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim N(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions.

Section A (60 marks)

1 Triangle ABC is shown in Fig. 1.

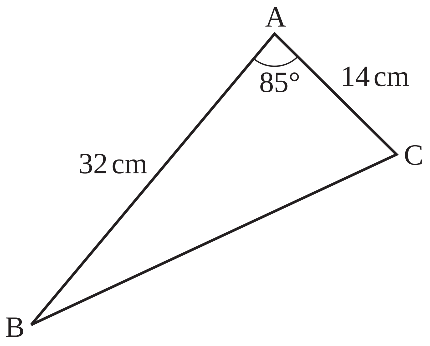


Fig. 1

Find the perimeter of triangle ABC. **[3]**

2 The curve $y = x^3 - 2x$ is translated by the vector $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$. Write down the equation of the translated curve. **[2]**

3 Fig. 3 shows a circle with centre O and radius 1 unit. Points A and B lie on the circle with angle AOB $= \theta$ radians. C lies on AO, and BC is perpendicular to AO.

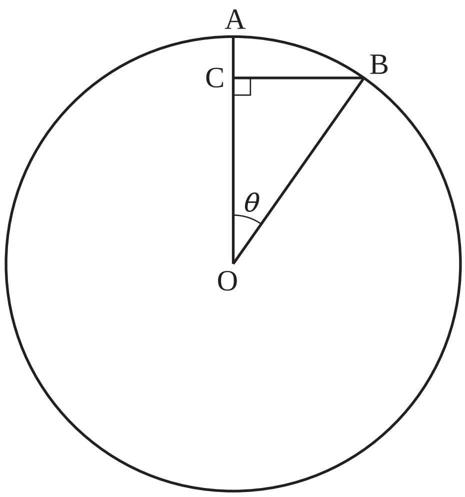


Fig. 3

Show that, when θ is small, $\text{AC} \approx \frac{1}{2}\theta^2$. **[2]**

4 **In this question you must show detailed reasoning.**

A curve has equation $y = x - 5 + \dfrac{1}{x-2}$. The curve is shown in Fig. 4.

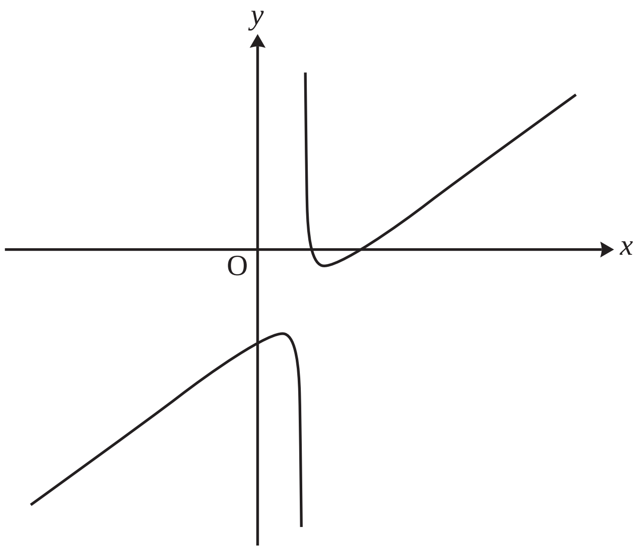


Fig. 4

(i) Determine the coordinates of the stationary points on the curve. **[5]**

(ii) Determine the nature of each stationary point. **[3]**

(iii) Write down the equation of the vertical asymptote. **[1]**

(iv) Deduce the set of values of x for which the curve is concave upwards. **[1]**

5 A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n, and the number of months after launch, t, can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

(i) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. **[2]**

(ii) Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

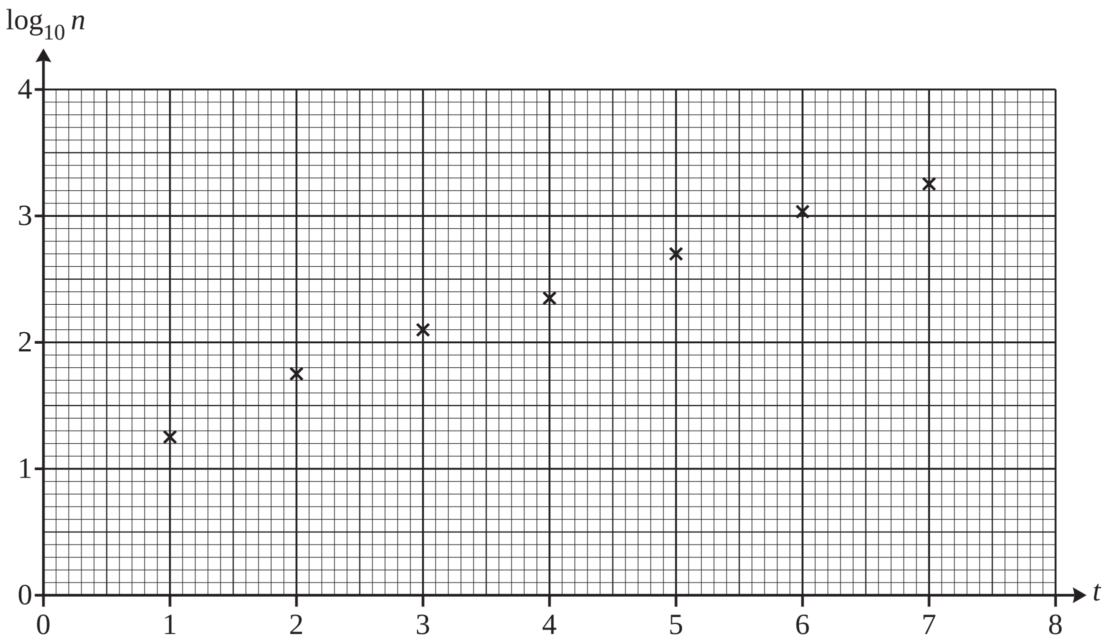


Fig. 5

Find estimates of the values of a and k. **[4]**

(iii) The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. **[4]**

(iv) Give a reason why the model may not be appropriate for large values of t. **[1]**

6 Find the constant term in the expansion of $\left(x^2 + \frac{1}{x}\right)^{15}$. **[2]**

7 **In this question you must show detailed reasoning.**

Fig. 7 shows the curve $y = 5x - x^2$.

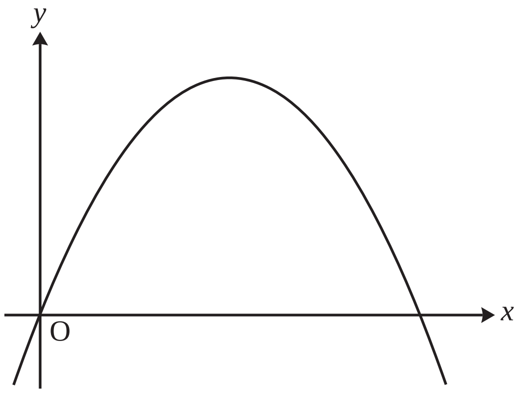


Fig. 7

The line $y = 4 - kx$ crosses the curve $y = 5x - x^2$ on the x-axis and at one other point.

Determine the coordinates of this other point. **[8]**

8 A curve has parametric equations $x = \dfrac{t}{1+t^3}$, $y = \dfrac{t^2}{1+t^3}$, where $t \neq -1$.

(i) **In this question you must show detailed reasoning.**

Determine the gradient of the curve at the point where $t = 1$. **[5]**

(ii) Verify that the cartesian equation of the curve is $x^3 + y^3 = xy$. **[3]**

9 The function $\mathrm{f}(x) = \dfrac{\mathrm{e}^x}{1-\mathrm{e}^x}$ is defined on the domain $x \in \mathbb{R}$, $x \neq 0$.

(i) Find $\mathrm{f}^{-1}(x)$. **[3]**

(ii) Write down the range of $\mathrm{f}^{-1}(x)$. **[1]**

10 Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where a and b can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with AC = AB.

(i) Show that $a - b + 1 = 0$. **[4]**

(ii) Determine the position vector of A such that triangle ABC has minimum area. **[6]**

Answer **all** the questions.

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

11 Line 8 states that $\frac{a+b}{2} \geqslant \sqrt{ab}$ for $a,\ b \geqslant 0$. Explain why the result cannot be extended to apply in each of the following cases.

(i) One of the numbers a and b is positive and the other is negative. **[1]**

(ii) Both numbers a and b are negative. **[1]**

12 Lines 5 and 6 outline the stages in a proof that $\frac{a+b}{2} \geqslant \sqrt{ab}$. Starting from $(a-b)^2 \geqslant 0$, give a detailed proof of the inequality of arithmetic and geometric means. **[3]**

13 Consider a geometric sequence in which all the terms are positive real numbers. Show that, for any three consecutive terms of this sequence, the middle one is the geometric mean of the other two. **[3]**

14 **(i)** In Fig. C1.3, angle CBD $= \theta$. Show that angle CDA is also θ, as given in line 23. **[2]**

(ii) Prove that $h = \sqrt{ab}$, as given in line 24. **[2]**

15 It is given in lines 31–32 that the square has the smallest perimeter of all rectangles with the same area. Using this fact, prove by contradiction that among rectangles of a given perimeter, $4L$, the square with side L has the largest area. **[3]**

END OF QUESTION PAPER

Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/03 Pure Mathematics and Comprehension
Printed Answer Booklet

Friday 15 June 2018 – Afternoon
Time allowed: 2 hours

You must have:
- Question Paper H640/03 (inserted)
- Insert (inserted)

You may use:
- a scientific or graphical calculator

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H64003

First name	
Last name	

Centre number						Candidate number				

INSTRUCTIONS
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Section A (60 marks)

1	
2	
3	

4(i)	
4(ii)	

4(iii)	
4(iv)	
5(i)	

5(ii)

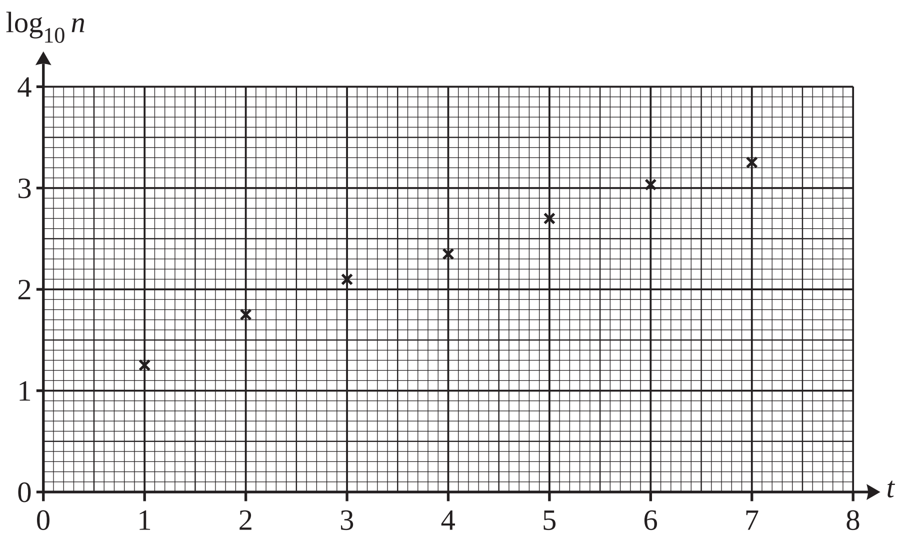

Fig. 5

5(iii)

5(iv)	
6	

7

8(i)	
8(ii)	
	(answer space continued on next page)

8(ii)	(continued)
9(i)	
9(ii)	

10(i)	

10(ii)	

Section B (15 marks)

The questions in this section refer to the article on the Insert. You should read the article before attempting the questions.

11 Line 8 states that $\frac{a+b}{2} \geqslant \sqrt{ab}$ for $a,\ b \geqslant 0$. Explain why the result cannot be extended to apply in each of the following cases.

(i) One of the numbers a and b is positive and the other is negative. **[1]**

(ii) Both numbers a and b are negative. **[1]**

11(i)

11(ii)

12 Lines 5 and 6 outline the stages in a proof that $\frac{a+b}{2} \geqslant \sqrt{ab}$. Starting from $(a-b)^2 \geqslant 0$, give a detailed proof of the inequality of arithmetic and geometric means. **[3]**

12	

13 Consider a geometric sequence in which all the terms are positive real numbers. Show that, for any three consecutive terms of this sequence, the middle one is the geometric mean of the other two. **[3]**

13	

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14 **(i)** In Fig. C1.3, angle CBD $= \theta$. Show that angle CDA is also θ, as given in line 23. **[2]**

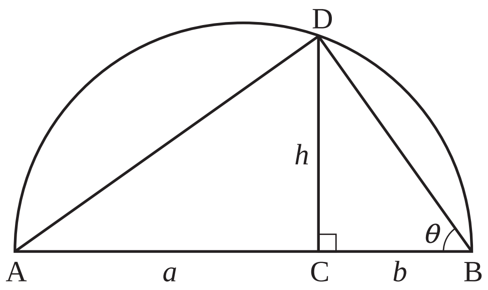

Fig. C1.3

(ii) Prove that $h = \sqrt{ab}$, as given in line 24. **[2]**

14(i)	

14(ii)	

15 It is given in lines 31–32 that the square has the smallest perimeter of all rectangles with the same area. Using this fact, prove by contradiction that among rectangles of a given perimeter, $4L$, the square with side L has the largest area. **[3]**

15	

ADDITIONAL ANSWER SPACE

If additional space is required, you should use the following lined page(s). The question number(s) must be clearly shown in the margin(s).

OCR
Oxford Cambridge and RSA

GCE

Mathematics B (MEI)

Unit **H640/03**: Pure Mathematics and Comprehension

Advanced Subsidiary GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner. If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case, please escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for *g*. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

k Anything in the mark scheme which is in square brackets […] is not required for the mark to be earned on this occasio, but shows what a complete solution might look like.

Question			Answer	Marks	AOs	Guidance	
1			$[BC^2] = 32^2 + 14^2 - 2 \times 32 \times 14 \cos 85°$	M1	3.1a	Use of cosine rule to find BC	1141.9…
			[BC] = 33.8 cm	A1	1.1		33.79…
			Perimeter = 79.8 cm	A1	1.1	Accept 80 provided 3 or more sf seen as BC	
				[3]			
2			$y = (x-1)^3 - 2(x-1) - 4$ oe	B1	1.1	Both factors $(x-1)$	$y = x^3 - 3x^2 + x - 3$ Must have y = for 2 isw after correct answer
				B1	1.1	- 4	
				[2]			
3			AC = [AO – OC] $= 1 - \cos\theta$ or $\cos\theta = 1 - AC$	M1	1.1a	**AG** Allow AC = AO – OC with $OC = \cos\theta$ for M1	
			θ small so AC $1 - \left(1 - \frac{\theta^2}{2}\right) = \frac{\theta^2}{2}$	E1	2.1	Convincing completion	
				[2]			
4	(i)		$\frac{dy}{dx} = 1 - \frac{1}{(x-2)^2}$	M1	1.1a	Attempt to differentiate with one term correct	
				A1	1.1	Correct derivative	
			$1 - \frac{1}{(x-2)^2} = 0$ at stationary points	M1	1.1a		
			$x - 2 = \pm 1$ so $x = 1, \quad 3$	A1	2.2a	Both values of x	
			$(1, -5)$ $(3, -1)$	A1	1.1	Both values of y - ft *their* x	
				[5]			

Question			Answer	Marks	AOs	Guidance	
4	(ii)		$\frac{d^2y}{dx^2} = \frac{2}{(x-2)^3}$	M1	1.1a		
			$x=3$ $\frac{d^2y}{dx^2} > 0$ (2) so minimum	A1	2.4		
			$x=1$ $\frac{d^2y}{dx^2} < 0$ (-2) so maximum	A1 [3]	2.4		
						OR Allow consideration of gradient either side of stationary point for **M1** Correct gradients above and below each tp **A1** Correct convincing conclusions (possibly with sketches) **A1**	
4	(iii)		$x=2$	B1 [1]	1.2		

x	f'(x)	x	f'(x)
0.5	0.56	2.5	-3.00
0.6	0.49	2.6	-1.78
0.7	0.41	2.7	-1.04
0.8	0.31	2.8	-0.56
0.9	0.17	2.9	-0.23
1	0.00	3	0.00
1.1	-0.23	3.1	0.17
1.2	-0.56	3.2	0.31
1.3	-1.04	3.3	0.41
1.4	-1.78	3.4	0.49
1.5	-3.00	3.5	0.56

Question			Answer	Marks	AOs	Guidance	
4	**(iv)**		$x > 2$	**A1** **[1]**	**2.2a**		FT *their* (iii) if region is to right of *their* x value
5	**(i)**		$\log_{[10]} n = \log_{[10]} a + kt \log_{[10]} 2$ This is of form $y = mx + c$ [with $\log_{[10]} n$ as y and t as x]	**M1** **E1** **[2]**	**1.1a** **1.1**	**AG** Allow $t \log 2^k$	
5	**(ii)**		Reasonable line of best fit drawn (by eye) Suitable method leading to a value eg use of intercept leading to $0.9 < \log a < 1.2$ So $7.4 < a < 15.85$ Suitable method leading to k value eg $k \log_{10} 2 = \text{gradient} \approx 0.33$ k in range $0 < k < 1.25$ and a in range $7.4 < a < 15.85$	**B1** **M1** **M1** **A1** **[4]**	**1.1a** **2.2a** **1.1** **2.2a**	With $0.9 < c < 1.2$ Finding gradient of line or sub'n of t and log n	May use 2 points from line or condone use of 2 given points If gradient of exactly 1/3 used $k = 1.10730936...$
5	**(iii)**		$500000 = 10 \times 2^{1.1t}$ $1.1t \log 2 = \log 50000$ $t = 14.2$ $t = 14$ is 1/3/18 So 1/4/18	**M1** **A1** **M1** **A1** **[4]**	**3.4** **1.1** **3.4** **3.2a**	Correct substitution Value of t (FT their a and k) Translation into date Rounding up	For k= 1.10730936.. t=14.1 Same answer
5	**(iv)**		Suitable reason	**E1**	**3.5b**		

Question			Answer	Marks	AOs	Guidance	
			e.g. The data are only for a short time scale and cannot extrapolate e.g. There will not be enough people for the growth to continue	[1]			
6			${}_{15}C_5\left(x^2\right)^5\left(\frac{1}{x}\right)^{10}$	M1	3.1a	Identifying term with $\left(x^2\right)^5\left(\frac{1}{x}\right)^{10}$	Must see
			3003	A1	1.1		
				[2]			
7						**DR**	
			$5x-x^2=x(5-x)$	M1	3.1a	Factorisation	
			$[x=0],\quad x=5$	A1	1.1	Finding 5	
			The line does not go through the origin so $x=5$	E1	2.4	Rejection of origin as a point where they cross	May be later
			$y=4-kx$ so $0=4-5k$	M1	3.2a		
			$k=\frac{4}{5}$	A1	1.1		
			$4-\frac{4}{5}x=5x-x^2$	M1	1.1		
			$x^2-5\frac{4}{5}x+4=0$ OR $5x^2-29x+20=0$				
			$(5x-4)(x-5)=0$	M1	1.1		
			$\left(\frac{4}{5},\ \frac{84}{25}\right)$ o.e.	A1	2.2a	$\frac{84}{25}=3\frac{9}{25}=3.36$	
				[8]			

Question			Answer	Marks	AOs	Guidance	
8	(i)		$\dfrac{dx}{dt}=\dfrac{(1+t^3)-t(3t^2)}{(1+t^3)^2}$	M1	1.1a	**DR** Use of quotient rule – may be gained for $\dfrac{dy}{dt}$ if $\dfrac{dx}{dt}$ not seen (allow ±)	
			$\dfrac{dx}{dt}=\dfrac{(1-2t^3)}{(1+t^3)^2}$ oe	A1	1.1		
			$\dfrac{dy}{dt}=\dfrac{2t(1+t^3)-t^2(3t^2)}{(1+t^3)^2}$				
			$\dfrac{dy}{dt}=\dfrac{(2t-t^4)}{(1+t^3)^2}$ oe	A1	1.1		
			$\dfrac{dy}{dx}=\dfrac{dy}{dt}\div\dfrac{dt}{dx}=\dfrac{2t-t^4}{1-2t^3}$	M1	1.1a	Substitution into *their* $\dfrac{dy}{dx}$ dep on earlier M1	
			$t=1\Rightarrow\dfrac{dy}{dx}=-1$	A1	2.1		
				[5]			
8	(ii)		LHS $=\dfrac{t^3+t^6}{(1+t^3)^3}$ oe	M1	1.1a	**AG** Expression for LHS	
			$=\dfrac{t^3(1+t^3)}{(1+t^3)^3}$	M1	1.1	Factorising	
			RHS $=\dfrac{t^3}{(1+t^3)^2}=$ LHS	A1	2.1	Completion of argument	
				[3]			

Question			Answer	Marks	AOs	Guidance	
9	(i)		$y=\dfrac{e^x}{1-e^x}$				
			$y(1-e^x)=e^x$	M1	1.1a	Clearing fractions	x and y may be interchanged at any stage
			$[\ y=e^x(1+y)\ ]\quad e^x=\dfrac{y}{1+y}$	A1	1.1	Expression for e^x	
			$f^{-1}(x)=\ln\left(\dfrac{x}{1+x}\right)$	A1 [3]	2.1	Condone '$y=$' Condone no brackets or mod	
9	(ii)		$f^{-1}(x)\neq 0$	B1 [1]	1.2	Allow $y\neq 0$ but not $x\neq 0$	
10	(i)		$\overrightarrow{AC}=\begin{pmatrix}2-a\\4-b\\2\end{pmatrix}$, $\overrightarrow{AB}=\begin{pmatrix}4-a\\2-b\\0\end{pmatrix}$	M1	1.1	Forming vectors for sides AB and AC	Implied by next M1
			$(4-a)^2+(2-b)^2=(2-a)^2+(4-b)^2+4$ o.e.	M1	1.1a	Use of AB = AC	
			$16-8a+a^2+4-4b+b^2=4-4a+a^2+16-8b+b^2+4$	M1	1.1	expanding	
			$4a-4b+4=0\Rightarrow a-b+1=0$	A1 [4]	2.1	**AG** Convincing completion	

Question			Answer	Marks	AOs	Guidance	
10	(ii)		D has position vector $\begin{pmatrix}3\\3\\1\end{pmatrix}$ where D is midpoint of BC	B1	3.1a	Midpoint OR if clearly minimising AC or AB - **M1** for relevant vector using *a* and *b* (May be implied by second M1)	
			$\overrightarrow{AD}=\begin{pmatrix}3-a\\2-a\\1\end{pmatrix}$	M1	1.1	Finding relevant vector in terms of *a* or *b* only	
			Area $=\frac{1}{2}AD.BC=\frac{2\sqrt{3}\sqrt{(3-a)^2+(2-a)^2+1}}{2}$	M1	1.1	Expression for AD or AD^2 (correct method but may have errors)	May use area proportional to AD, AC or AB without calculation of expression for area
			$\sqrt{3}\sqrt{2\left((a-2.5)^2+0.75\right)}$	M1	3.1a	Completion of square	Or differentiation of AD, AD^2, AC, AB, AC^2 or AB^2.
			$a=2.5$ for min	A1	2.2a		
			Position vector $\begin{pmatrix}2.5\\3.5\\0\end{pmatrix}$	A1	3.2a		
				[6]			
11	(i)		The geometric mean cannot be calculated	E1 [1]	2.3		

Question			Answer	Marks	AOs	Guidance	
11	(ii)		The arithmetic mean will be less than the geometric mean	E1 [1]	2.3	E.g. The arithmetic mean will be negative	
12			$(a-b)^2 \geq 0 \Rightarrow a^2 - 2ab + b^2 \geq 0$ $a^2 + b^2 \geq 2ab \Rightarrow a^2 + 2ab + b^2 \geq 4ab$ $(a+b)^2 \geq 4ab$ $a + b \geq \sqrt{4ab} \Rightarrow a + b \geq 2\sqrt{ab} \Rightarrow \frac{a+b}{2} \geq \sqrt{ab}$	B1 B1 B1 [3]	1.1 3.1a 2.1	Squaring bracket Adding $2ab$ to each side Square root and correct completion	
13			Let the terms be $\frac{c}{r}$, c, cr The geometric mean of first and last is $\sqrt{\frac{c}{r}cr}$ $\sqrt{\frac{c}{r}cr} = \sqrt{c^2} = c$; this is the middle term	B1 M1 E1 [3]	3.1a 1.1 2.1	Expressions for three consecutive terms of a GP (any correct form) Expression for GM of first and last term (any correct form) *FT* their terms **AG** Correct completion	
14	(i)		Angle BDC = $90 - \theta$ (angles of triangle) Angle CDA = θ (Angle ADB = 90° as it is the angle in a semicircle)	M1 E1 [2]	2.1 2.2a	Including reason Answer given so mark is for reason	Reasons can be given in either order

Question			Answer	Marks	AOs	Guidance	
	(ii)		Triangle ACD, $\tan\theta = \frac{h}{b}$ Triangle BCD, $\tan\theta = \frac{a}{h}$	M1	1.1	At least one correct expression for tan θ	Alternative method: triangle ACD is similar to triangle DBC
			$\frac{a}{h} = \frac{h}{b} \Rightarrow h^2 = ab \Rightarrow h = \sqrt{ab}$	E1 [2]	2.1	Setting expressions equal and correct completion to given answer **AG**	
15			Suppose that for the given perimeter, $4L$, there is a rectangle which is larger in area than the square.	M1	2.5	Setting up a statement for contradiction.	
			There is a square which has the same area as this rectangle but a smaller perimeter so its side is less than L.	A1	3.1a	Use of statement in line 31-32	
			The square with side L has perimeter $4L$ and an area larger than the given rectangle. This is a contradiction so the square must have the largest area of all rectangles with given perimeter.	A1 [3]	2.2a	Completion to correct conclusion, including contradiction	

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Examiners' report

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/03 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H640/03 series overview

This was the third and final paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates. This paper contributes 27.3% of the total A-level and assesses content solely from pure mathematics.

To do well in this component, candidates need to be able to apply their knowledge of the syllabus content in a variety of modelling contexts and to make efficient use of calculator technology.

Section A overview

This section contained questions on pure maths of a range of length and difficulty.

Question 1

1 Triangle ABC is shown in Fig. 1.

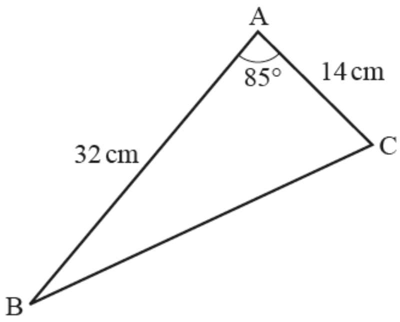

Fig. 1

Find the perimeter of triangle ABC. **[3]**

For the majority of candidates this provided a straightforward start to the paper. The few candidates who did not score full marks either misread the question and only found the length of *BC*, did not give sufficient accuracy in their answer. A small minority of candidates did not recall accurately the Cosine Rule.

AfL

This question is in degrees, but many questions at A Level involve the use of radians. It is important that candidates are confident switching between units on their calculators. The specification advice to explicitly write down any expressions to be evaluated by calculator would ensure partial credit where the incorrect setting on the calculator is used.

Question 2

2 The curve $y = x^3 - 2x$ is translated by the vector $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$. Write down the equation of the translated curve. **[2]**

Those candidates who knew to substitute *x*-1 generally scored fully here.

AfL

Note that the question did not require the answer to be expanded and some candidates wasted time doing this. Whilst this is an important skill, if required the question will specify a specific format for the answer.

Question 3

3 Fig. 3 shows a circle with centre O and radius 1 unit. Points A and B lie on the circle with angle AOB $= \theta$ radians. C lies on AO, and BC is perpendicular to AO.

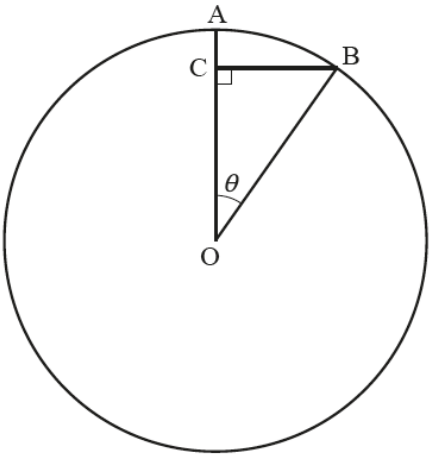

Fig. 3

Show that, when θ is small, $\mathrm{AC} \approx \frac{1}{2}\theta^2$. **[2]**

This is the first example of a 'Show that' question in this paper and candidates could not score unless they explained the logic of their initial expression

Exemplar 1

$OA = 1$

$CO = \cos\theta \approx 1 - \frac{\theta^2}{2}.$

$1 - 1 + \frac{\theta^2}{2} = CA$

$\theta^2/2 = AC$

The candidate knows the small angle approximation to use but does not explain where their first equation for *CA* comes from. Therefore no marks can be earned.

Question 4 (i)

4 In this question you must show detailed reasoning.

A curve has equation $y = x - 5 + \frac{1}{x-2}$. The curve is shown in Fig. 4.

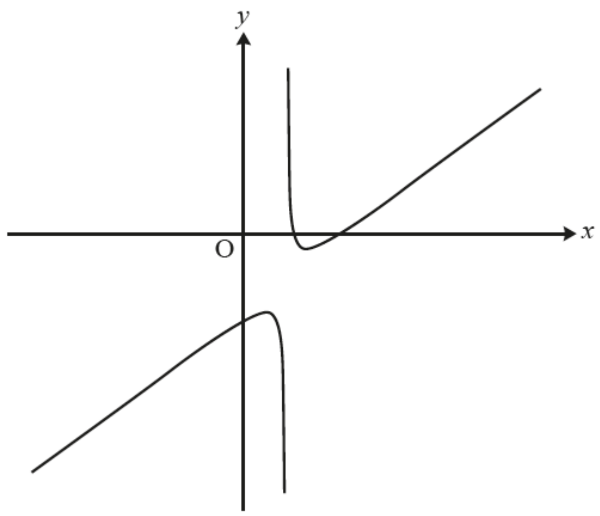

Fig. 4

(i) Determine the coordinates of the stationary points on the curve. **[5]**

Most candidates were able to score full marks here following correct differentiation and solution of what ended up as a quadratic equation. Many different ways were used to solve the equation usually without any wrong working. Examiners were pleased to see correct notation used in this question.

Question 4 (ii)

(ii) Determine the nature of each stationary point. **[3]**

Again most candidates were successful in classifying the stationary points with use of the second derivative being the most common method. A few considered the gradient either side of each turning point and then reasoned their way to a correct conclusion.

Question 4 (iii)

(iii) Write down the equation of the vertical asymptote. **[1]**

There appeared to be confusion as to the meaning of vertical asymptote which led to a low success rate for this part.

Question 4 (iv)

(iv) Deduce the set of values of x for which the curve is concave upwards. **[1]**

As in part (iii), a large proportion of the candidates struggled with this part.

AfL The OCR B (MEI) H640 specification defines the terms "concave upwards" and "concave downwards" as those that will be used in examination questions.

5 A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n, and the number of months after launch, t, can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

(i) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. **[2]**

Question 5 (i)

Most candidates could manage the use of logs and rearranging but quite a number did not finish off and explain why it was a straight line.

Exemplar 2

$n = a2^{kt}$

$\log_{10} n = \log_{10} a2^{kt}$

$= \log_{10} a + \log_{10}(2^k)^t = \log_{10} a + t\log_{10} 2^k$

The candidate correctly rearranges but does not relate their equation to $y = mx + c$ so only scores 1.

Exemplar 3

$n = a \times 2^{kt}$

~~$\log n = \log (a$~~

$\log_{10} n = \log_{10} (a \times 2^{kt})$

$\log_{10} n = \log_{10} a + \log_{10} 2^{kt}$

$\log_{10} n = \log_{10} a + t \log_{10} 2^{k}$

$y \qquad c \qquad x \qquad m.$

forms equation of straight line graph = $y = mx + c$.

~~an~~ $y = a^{kx}$

$\log y = \log x + \log a.$

$y = kx^n$

$\log y = \log(kx^n)$

$\log y = \log k + \log x^n$

$\log y = \log k + n \log x$

The candidate makes doubly sure of the final mark by saying it is *y* = *mx* + *c* and showing which parts of their equation correspond to *m, c* etc.

Question 5 (ii)

(ii) Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

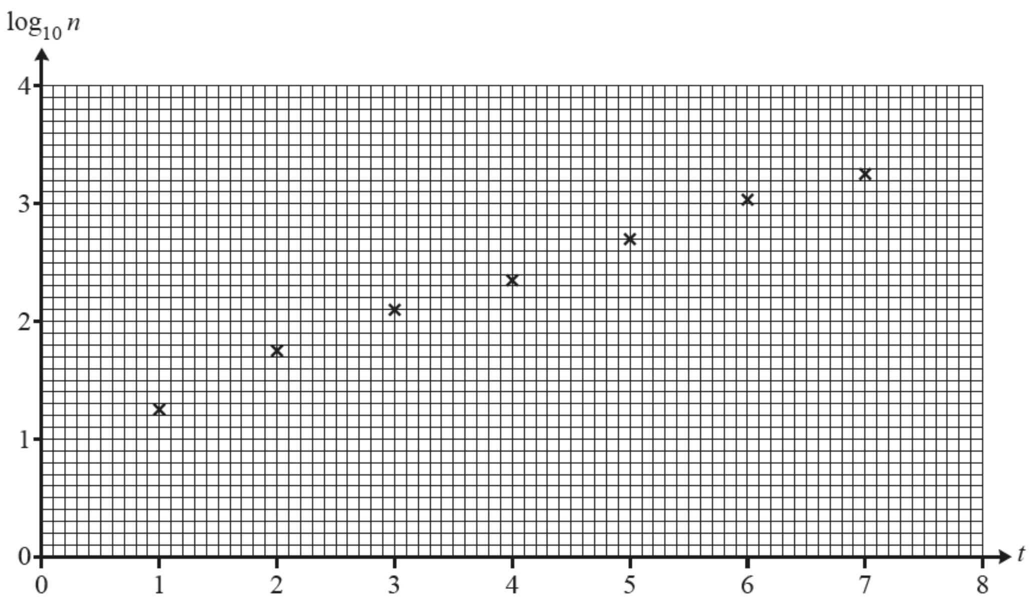

Fig. 5

Find estimates of the values of a and k. **[4]**

Examiners were surprised how few actually drew in a line of best fit. There is the possibility that candidates drew their line of best fit on the question paper, rather than the answer booklet. Candidates should be reminded that it is only the answer booklet that is seen by the examiner, therefore any rough work, sketches, diagrams or annotations that are drawn on the question paper will not gain credit.

AfL Choosing two of the points to determine the gradient is not generally the best method but finding the gradient of their line is. It is possible that a candidate may have determined that all the points were generally close to the 'line'. This assumption was not penalised in this assessment.

Question 5 (iii)

(iii) The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. **[4]**

Many started well but many candidates did not correctly translate their *t* into the correct date or round up.

AfL One of the features of the reformed qualification criteria is the increased emphasis on modelling. Candidates should ensure that their final answer reflects the context of the original problem and not only focus on the mathematical techniques.

Question 5 (iv)

(iv) Give a reason why the model may not be appropriate for large values of t. **[1]**

A suitable reason was generally supplied for this part, even if the previous part was not correct.

AfL Candidates should be reminded that subsequent questions related to the model may not require a successful answer to previous parts using the model. Good exam practice involves reading the full question before answering the individual parts, especially if the first parts appear challenging.

Question 6

6 Find the constant term in the expansion of $\left(x^2+\frac{1}{x}\right)^{15}$. **[2]**

This was a standard question for which many fully correct solutions were provided. Whilst not required, a clear justification of why $^{15}C_5$ is needed may help ensure the correct answer of 3003 is obtained, and could gain partial credit if a minor error is made.

Question 7

7 In this question you must show detailed reasoning.

Fig. 7 shows the curve $y = 5x - x^2$.

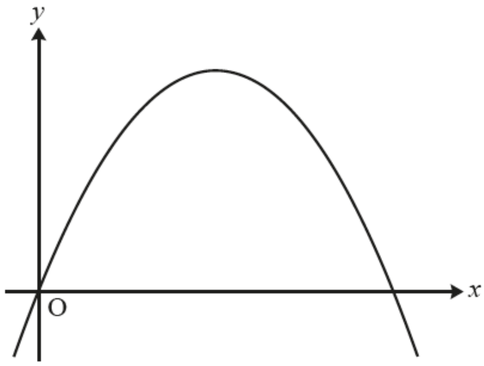

Fig. 7

The line $y = 4 - kx$ crosses the curve $y = 5x - x^2$ on the x-axis and at one other point.

Determine the coordinates of this other point. **[8]**

This was a question where candidates generally understood the necessary steps required. Some solutions lost a mark because they did not show why x = 0 should be rejected in favour of x = 5 being used. Also a few candidates got a little bogged down by focussing on the discriminant.

Question 8 (i)

8 A curve has parametric equations $x = \dfrac{t}{1+t^3}$, $y = \dfrac{t^2}{1+t^3}$, where $t \neq -1$.

(i) In this question you must show detailed reasoning.

Determine the gradient of the curve at the point where $t = 1$. **[5]**

This part saw many attempts at the correct process, but there were some mistakes seen in applying the quotient rule generally with confusion over signs. More difficulty was experienced by those who chose to use the product rule but did not appreciate the need to apply the chain rule to the differentiation of $(1+t^3)^{-1}$.

AfL

Possibly the use of the product rule to prove the quotient rule leads some candidates to assume that that is the method to use to differentiate quotients. Candidates should know the difference and be able to apply both the product rule and the quotient rule accurately as appropriate.

Question 8 (ii)

(ii) Verify that the cartesian equation of the curve is $x^3 + y^3 = xy$. **[3]**

This part was generally started well, but many candidates struggled to complete the argument to score fully.

The majority tackled this question by starting with finding an expression for $x^3 + y^3$ in terms of t, but a few chose to use the expression for x or y and make t the subject then continue to eliminate t from the expressions and generally complete the argument satisfactorily.

Exemplar 4

$$x = \frac{t}{1+t^3} \qquad y = \frac{t^2}{1+t^3}$$

$$\frac{1}{y} = \frac{1+t^3}{t^2}$$

$$\frac{x}{y} = \frac{t}{1+t^3} \times \frac{1+t^3}{t^2}$$

$$= \frac{t}{t^2} = t^{-1} = \frac{1}{t}$$

$$t = \frac{y}{x}$$

$$x = \frac{\frac{y}{x}}{1+\left(\frac{y}{x}\right)^3} \qquad x\left(1+\left(\frac{y}{x}\right)^3\right) = \frac{y}{x}$$

$$x + \frac{xy^3}{x^3} = \frac{y}{x}$$

$$x^4 + xy^3 = x^2y$$

divide by x $\qquad \cancel{x^3 + y^3 = xy}$

$$x^3 + y^3 = xy$$

This candidate does full proof of the result gaining all 3 marks.

Exemplar 5

$$\left(\frac{t}{1+t^3}\right)^3 + \left(\frac{t^2}{1+t^3}\right)^3 = \frac{t^3}{(1+t^3)^3} + \frac{t^6}{(1+t^3)^3}$$

$$= \frac{t^6+t^3}{(1+t^3)^3} = \frac{t^3(t^3+1)}{(1+t^3)^3} = \frac{t^3}{(1+t^3)^2} = xy$$

Because $xy = \left(\frac{t}{1+t^3}\right)\left(\frac{t^2}{1+t^3}\right) = \frac{t^3}{(1+t^3)^2}$

$$\frac{dx}{dt} = (1+t^3)^{-1} - 3t^3(1+t^3)^{-2}$$

$$\frac{dy}{dt} = 2t(1+t^3)^{-1} - 3t^4(1+t^3)^{-2}$$

$t=1$, $\frac{dx}{dt} = -\frac{1}{4}$

$$\frac{dy}{dt} = \frac{1}{4}$$

$$\frac{dy}{dx} = \frac{1}{4} \div -\frac{1}{4} = -1$$

This candidate shows appropriate initial working to verify the equation, but misses out on the final mark, as they do not state what they have shown.

Question 9 (i)

9 The function $f(x) = \frac{e^x}{1-e^x}$ is defined on the domain $x \in \mathbb{R}$, $x \neq 0$.

(i) Find $f^{-1}(x)$. **[3]**

Most candidates were well prepared for this standard question. Errors seen tended to be in forgetting to interchange *x* and *y*.

Question 9 (ii)

(ii) Write down the range of $f^{-1}(x)$. **[1]**

A common error here was to state that $f^{-1}(x) \neq 0$.

Question 10 (i)

10 Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where a and b can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with AC = AB.

(i) Show that $a - b + 1 = 0$. **[4]**

This part was generally answered well.

Question 10 (ii)

(ii) Determine the position vector of A such that triangle ABC has minimum area. **[6]**

This non-standard problem led to many different approaches but finding the midpoint of BC seemed the most logical start. The need to find a relevant vector such as A to the midpoint of *BC* in terms of one variable was appreciated by most candidates. The challenging part of this question was to find *a* for the minimum area and then use it to find the correct vector.

Section B overview

This section consisted of questions testing comprehension of an article on arithmetic and geometric means and their different properties. It is recommended that candidates read the full article before attempting to answer any of the questions.

Question 11 (i)

11 Line 8 states that $\frac{a+b}{2} \geqslant \sqrt{ab}$ for $a,\ b \geqslant 0$. Explain why the result cannot be extended to apply in each of the following cases.

(i) One of the numbers a and b is positive and the other is negative. **[1]**

This was answered well by the majority of candidates.

Question 11 (ii)

(ii) Both numbers a and b are negative. **[1]**

This was also answered well.

Question 12

12 Lines 5 and 6 outline the stages in a proof that $\frac{a+b}{2} \geqslant \sqrt{ab}$. Starting from $(a-b)^2 \geqslant 0$, give a detailed proof of the inequality of arithmetic and geometric means. **[3]**

Many fully correct proofs seen here. However some candidates stated that $a^2 + b^2 = (a + b)^2$.

Question 13

13 Consider a geometric sequence in which all the terms are positive real numbers. Show that, for any three consecutive terms of this sequence, the middle one is the geometric mean of the other two. **[3]**

Most candidates were able to give general expressions for three consecutive terms of a geometric series and their response progressed successfully. However, some candidates lost a mark by not correctly completing their explanation by relating to the middle term.

Question 14 (i)

14 **(i)** In Fig. C1.3, angle CBD $= \theta$. Show that angle CDA is also θ, as given in line 23. **[2]**

This question drew on prior knowledge from GCSE. Only few managed to give both 'angles in a triangle' and 'angle in a semicircle'.

Question 14 (ii)

(ii) Prove that $h = \sqrt{ab}$, as given in line 24. **[2]**

This was generally proved correctly.

Question 15

15 It is given in lines 31–32 that the square has the smallest perimeter of all rectangles with the same area. Using this fact, prove by contradiction that among rectangles of a given perimeter, $4L$, the square with side L has the largest area. **[3]**

Proof by contradiction was challenging for the majority of candidates. Many offered no solution and some tried a proof by deduction. Those candidates that were successfully were able to make progress from a clear, suitable initial statement for contradiction.

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Exemplar Candidate Work

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/03 Summer 2018 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification http://www.ocr.org.uk/Images/308740-specification-accredited-a-level-gce-mathematics-b-mei-h640.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1

1 Triangle ABC is shown in Fig. 1.

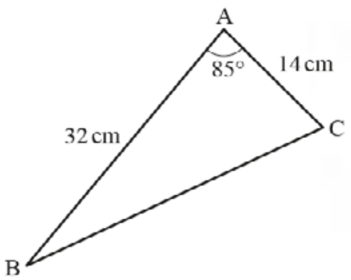


Fig. 1

Find the perimeter of triangle ABC. **[3]**

Exemplar 1 — 3 marks

1

$BC = \sqrt{32^2 + 14^2 - 2 \times 32 \times 14 \cos 85} = 33.79$

$33.79 + 32 + 14 = 79.8\text{cm}$

Examiner commentary

High ability candidates could cope with the two-stage method of this question and give their answer to appropriate accuracy.

Exemplar 2 — 2 marks

1

$a^2 = 14^2 + 32^2 - 2(14)(32)(\cos 85)$

$a^2 = 1141.908454$

$a = 33.792$ M1

$= 33.8$ cm A1

Examiner commentary

Lower ability candidates could often apply cosine rule correctly but then believed they had finished the question.

Exemplar 3 1 mark

1 $a^2 = b^2 + c^2 - 2bc \cos A$

$a^2 = 32^2 + 14^2 - 2 \times 32 \times 14 \times \cos 85$ M1

$a^2 = 1086.128779$

$a = 32.95646794$ A0

$32 + 32.95646794 + 14 = \cancel{78}.$

$= 78.95646794$

$= 78.96$ (2dp) cm A0

Examiner commentary

By showing their working, this candidate was able to gain the initial mark even though they made an error entering the data into the calculator.

Question 2

2 The curve $y = x^3 - 2x$ is translated by the vector $\begin{pmatrix} 1 \\ -4 \end{pmatrix}$. Write down the equation of the translated curve. **[2]**

Exemplar 1 — 1 mark

2	$y = (x+1)^3 - 2x - 4$ B1

Examiner commentary

The highest ability candidates had no trouble with this question and avoided any temptation to expand brackets for the x translation but those who were less secure tended to only manage 1 mark for the y translation.

Question 3

3 Fig. 3 shows a circle with centre O and radius 1 unit. Points A and B lie on the circle with angle AOB = θ radians. C lies on AO, and BC is perpendicular to AO.

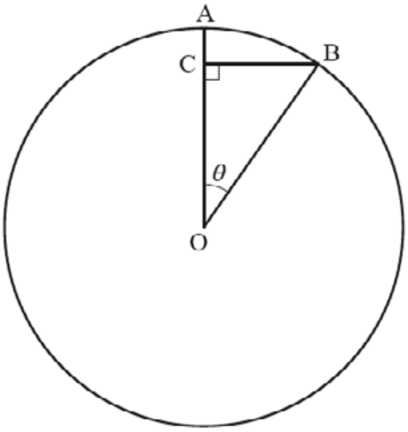


Fig. 3

Show that, when θ is small, AC $\approx \frac{1}{2}\theta^2$. **[2]**

Exemplar 1 — 2 marks

3

$OA = OB = 1$

$OC = 1\cos\theta$

$= 1(1 - \tfrac{1}{2}\theta^2)$

$= 1 - \tfrac{1}{2}\theta^2$

$OA = 1$

so $AC = OA - OC$

$= 1 - (1 - \tfrac{1}{2}\theta^2)$

$= \tfrac{1}{2}\theta^2$

Examiner commentary

Whilst many of the lower ability candidates did not make any headway with this question, some of the medium ability could manage to score full marks if they showed their reasoning clearly here with AC = OA – C and the small angle substitution for cosθ evident.

Question 4 (i), (ii), (iii) and (iv)

4 In this question you must show detailed reasoning.

A curve has equation $y = x - 5 + \frac{1}{x-2}$. The curve is shown in Fig. 4.

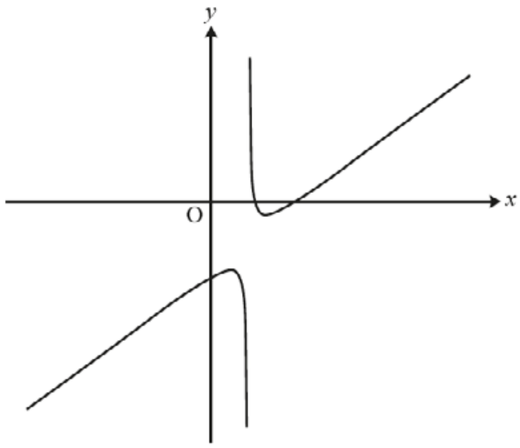


Fig. 4

(i) Determine the coordinates of the stationary points on the curve. **[5]**

(ii) Determine the nature of each stationary point. **[3]**

(iii) Write down the equation of the vertical asymptote. **[1]**

(iv) Deduce the set of values of x for which the curve is concave upwards. **[1]**

Exemplar 1 — 5 marks

(i)

4(i)

$y = x - 5 + (x-2)^{-1}$ $\quad u = (x-2)$

$\frac{dy}{dx} = 1 - (x-2)^{-2}$ $\quad \frac{du}{dx} = 1$

$\frac{dy}{dx} = 0 \quad 0 = 1 - \frac{1}{(x-2)^2}$

$1 = \frac{1}{(x-2)^2}$

$(x-2)^2 = 1$

$x - 2 = \pm 1$

$x = 2 \pm 1$

$x = 3$ or $x = 1$

$y = -1$ $\quad y = -5$

$(3, -1)$ or $(1, -5)$

(ii) 3 marks

4(ii)	$\frac{d^2y}{dx^2} = 2(x-2)^{-3}$
	$x=3, \frac{d^2y}{dx^2} = 2 \quad 2>0 \Rightarrow$ minimum
	$x=1, \frac{d^2y}{dx^2} = -2 \quad -2<0 \Rightarrow$ maximum

(iii) 1 mark

4(iii)	$x=2$

(iv) 1 mark

4(iv)	$\frac{d^2y}{dx^2} > 0$
	$\frac{2}{(x-2)^3} > 0$
	$x > 2$

Examiner commentary

The best candidates had little problem with this question correctly differentiating, equating to 0 and solving for the stationary points and most usually, and successfully, using the second derivative to classify them.

Exemplar 2 1 mark

(i)

4(i)	Stationary points when $\frac{dy}{dx} = 0$
	$y = x - 5 + \frac{31}{x-2}$
	$\frac{dy}{dx} = 1 + \cancel{x}$ M1

Examiner commentary

This candidate earned a method mark for attempting to differentiate and the '1' is correct for x (or $x - 5$). However, they could not then go on to score any more on this question. Candidates need a secure grounding in basic calculus techniques to access a wide range of questions in this qualification.

Question 5 (i)

5 A social media website launched on 1 January 2017. The owners of the website report the number of users the site has at the start of each month. They believe that the relationship between the number of users, n, and the number of months after launch, t, can be modelled by $n = a \times 2^{kt}$ where a and k are constants.

(i) Show that, according to the model, the graph of $\log_{10} n$ against t is a straight line. **[2]**

Exemplar 1 — 2 marks

5(i)

$n = a \times 2 \cancel{k}^{kt}$

$\log_{10} n = \log a2^{kt}$

$\log_{10} n = \log a + \log 2^{kt}$

$\log_{10} n = \log_{10} a + kt \log_{10} 2$

$\log_{10} n = (k \log_{10} 2)t + \log_{10} a$

$\downarrow \quad \downarrow \quad \downarrow \quad \downarrow$

$y = m \quad x + c$

gradient (m) is constant therefore it is a straight line

Examiner commentary

Nearly all candidates could use logs to get a correct linear equation. This candidate used very clear working (their arrows) to show how their equation corresponded to $y = mx + c$.

Question 5 (ii)

5 **(ii)** Fig. 5 shows a plot of the values of t and $\log_{10} n$ for the first seven months. The point at $t = 1$ is for 1 February 2017, and so on.

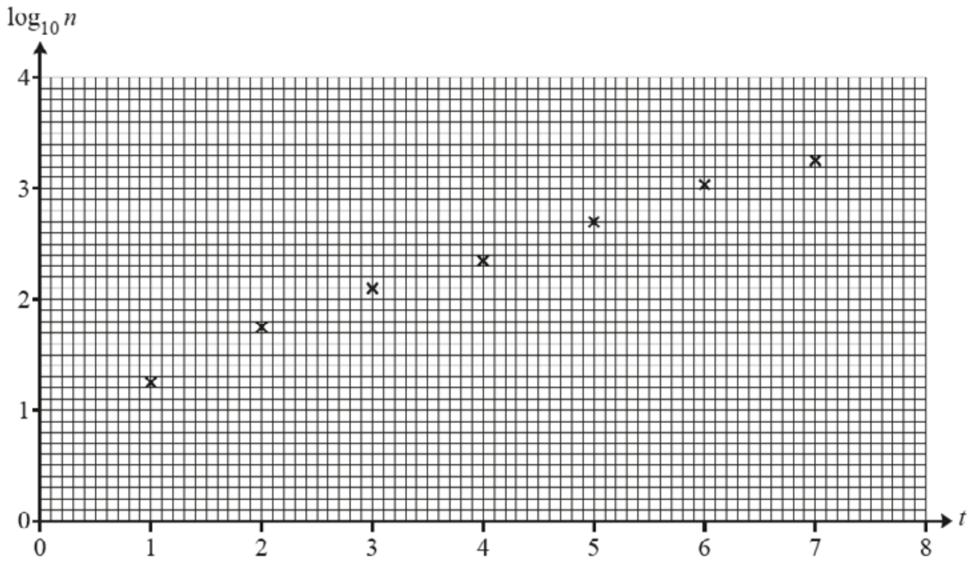


Fig. 5

Find estimates of the values of a and k. **[4]**

Exemplar 1 3 marks

5(ii)

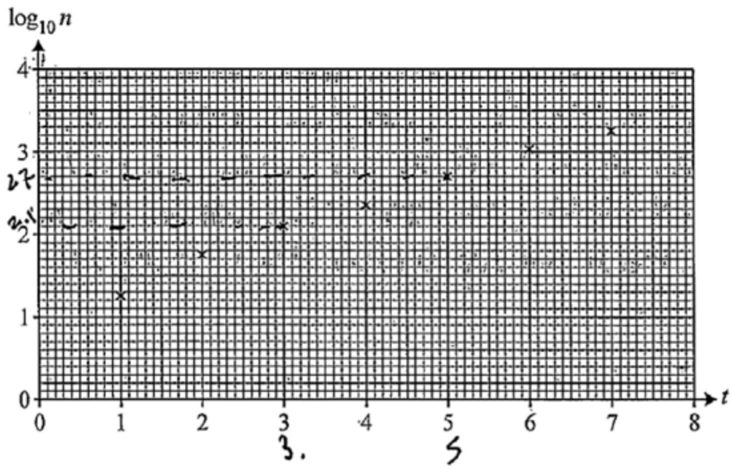


Fig. 5

B0

When $t=3$ $\log_{10} n = 2.1.$

$\log_{10} = \log$ in my writing.

$2.1 = (k\log 2)t + \log a$ If $k = 0.9966$

When $t=5$ $\log n = 2.7$: $2.7 = 5\log_2 + \log a$ ($\times 0.9966$)

$2.7 = (k\log 2)t + \log a$ $\log a = 1.1997$

$2.7 = 5\log_2 k + \log a$ $a = 10^{1.1997}$

$2.1 = 3\log_2 k + \log a$ ~~a = 15.66~~

~~0.42t~~ ~~0.27~~ $0.6 = 2\log_2 k$ $a = 15.85.$

$k = \cancel{4}\ 0.9966$

Examiner commentary

This is a good example of a candidate that knows essentially what they are doing but just needs a few pointers to end up with a perfect solution. The candidate calculated the gradient from two of the given points with no consideration of how representative they might be. To take all the data points into consideration a line of best fit should be drawn (this alone would have earned the 4th mark) and then the gradient of that line calculated.

Question 5 (iii)

5 **(iii)** The owners of the website wanted to know the date on which they would report that the website had half a million users. Use the model to estimate this date. **[4]**

Exemplar 1 — 2 marks

5(iii)

$500{,}000 = 15.85 \times 2^{0.9966t}$

$31550 = 2^{0.9966t}$ M1

$\log_2 31550 = 0.9966t$

$14.945 = 0.9966t$

$t = 14.996$ A1

$t = 15$ months.

Examiner commentary

As in this example, most candidates could use their values of k and a to find a sensible number of months. However only the highest ability had read and appreciated the need to answer as a date and then to round up to the next 1st of the month.

Question 5 (iv)

5 **(iv)** Give a reason why the model may not be appropriate for large values of t. **[1]**

Exemplar 1 — 1 mark

5(iv) The number of users will increase to the absolute maximum amount of people on earth. The number would be less as not everyone has internet access. As well as this some people could have two or more accounts to use the website therefore in the long term (larger values of t) the model won't be appropriate

Examiner commentary

Very few candidates did not feel able to provide an answer to this question; even the lowest ability candidates were able to gain the mark here where an attempt was made, regardless of the success on the previous parts.

Question 6

6 Find the constant term in the expansion of $\left(x^2+\frac{1}{x}\right)^{15}$. **[2]**

Exemplar 1 — 2 marks

6 $\left(x^2+\frac{1}{x}\right)^{15} = \sum_{r=0}^{15}\binom{15}{r} x^{2r}\, x^{-(15-r)}$

x^0 is constant $(=1)$

$x^{2r} \cdot x^{-(15-r)} = 0$

$2r-(15-r)=0$

$3r-15=0$ ✓

$r=5$

const. term $=\binom{15}{5}\cdot x^0 = 3003$ ✓

Examiner commentary

This candidate has set out their thinking with a clear and concise mathematical argument. Whilst this level of reasoning was not required for this question, this would have helped secure partial credit had a minor error been made.

Question 7

7 **In this question you must show detailed reasoning.**

Fig. 7 shows the curve $y = 5x - x^2$.

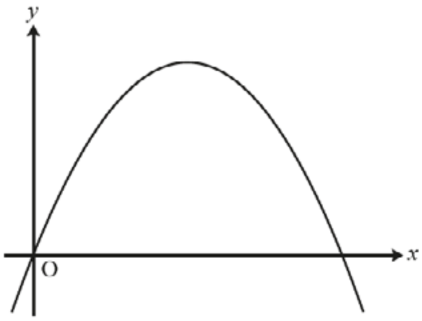


Fig. 7

The line $y = 4 - kx$ crosses the curve $y = 5x - x^2$ on the x-axis and at one other point.

Determine the coordinates of this other point. **[8]**

Exemplar 1 — 5 marks

7

$4 - kx$ crosses $5x - x^2$

$4 - kx = 5x - x^2$

$x^2 + (-5-k)x + 4 = 0$

↑ they cross at x-axis where $y = 0$

$5x - x^2 = 0$

$x^2 = 5x$ M0 ^ ^

$x = 5$

$x^2 + (-5-k)x + 4$ has a root at $x = 5$

$25 + 5(-5-k) + 4 = 0$

$-5(5+k) = -29$

$5 + k = \frac{29}{5}$ M1

$k = 29/5 - 5$ A1

$= 4/5$

M1

$x^2 + (-5 - 4/5)x + 4 = 0 \Rightarrow x^2 - 5.8x + 4 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ M1

we already knew about $x = 5$

$x = \frac{5.8 \pm \sqrt{5.8^2 - 4(4)}}{2} = \boxed{5}\ 0.8$ ✓ $x = 0.8$

$4 - 0.8(0.8) = 3.36 = y$

$(0.8, 3.36)$ ✓ A1

Examiner commentary

This question made use of the "**In this question you must show detailed reasoning**". Some otherwise very good candidates lost up to three marks (as is the case here) as they did not provide justification for using $x = 5$. A full detailed response needed to include the factorisation of their equation ($x^2 - 5x = 0$) to get the two potential crossing points (x = 5 and 0) and then reject the origin as being inadmissible.

Question 8 (i)

8 A curve has parametric equations $x = \frac{t}{1+t^3}$, $y = \frac{t^2}{1+t^3}$, where $t \neq -1$.

(i) **In this question you must show detailed reasoning.**

Determine the gradient of the curve at the point where $t = 1$. **[5]**

Exemplar 1 — 2 marks

8(i)

$x = \frac{t}{1+t^3}$, $y = \frac{t^2}{1+t^3}$, $t \neq 1$

$\frac{dx}{dt}$ $\cancel{\times \frac{dt}{}}$ $\frac{dy}{dt} \div \frac{dx}{dt} = \frac{dy}{dx}$

$\frac{dx}{dt} = (t)(1+t^3)^{-1}$ $fx = t$ $f'x = 1$ $gx = (1+t^3)^{-1}$ $g'x = -(1+t^3)^{-2}$

$= -t(1+t^3)^{-2} + (1+t^3)^{-1}$ M1 A0

$\frac{dy}{dt} = (t^2)(1+t^3)^{-1}$ $fx = t^2$ $f'x = 2t$ $gx = (1+t^3)^{-1}$ $g'x = -(1+t^3)^{-2}$

$= -t^2(1+t^3)^{-2} + 2t(1+t^3)^{-1}$

$\frac{dy}{dx} =$ M0 $\frac{2t(1+t^3)^{-1} - t^2(1+t^3)^{-2}}{-t(1+t^3)^{-2} + (1+t^3)^{-1}} = \boxed{3}$ M1

Examiner commentary

This script shows the common error in answering this question. The parametric equations in the question were both given as straightforward quotients (e.g. $x = \frac{t}{1+t^3}$) and examiners hoped that candidates would therefore use the quotient rule to find $\frac{dx}{dt}$ and $\frac{dy}{dt}$. However, as in this example, a number of candidates considered the quotient as a product $\left((t)(1+t^3)^{-1}\right)$ and whilst this is not wrong in any way, lower ability candidates could not use the chain rule within the product rule generally forgetting to multiply by the differential of the bracket.

It could be that this stems from candidates being shown the proof of quotient rule and then using the proof method rather than the quotient rule itself.

Question 8 (ii)

8 **(ii)** Verify that the cartesian equation of the curve is $x^3 + y^3 = xy$. **[3]**

Exemplar 1 2 marks

8(ii)

$y = tx$

$x = t/1+t^3$ find t in terms of x

$x + \cancel{tt^3}\ xt^3 = t$ $t^3(1/t^2 - x) = x$

~~tell~~ $x = t - \cancel{\ldots}\ xt^3$ $t^3 = \frac{xt^2}{1-x}$ *

$\cancel{x = \ldots}$ $\cancel{t = \ldots}$

8(ii) (continued)

$y = (\frac{x}{1-x^3})x$ $y = \frac{x}{\sqrt[3]{1-x}}$

$\cancel{y - yx^3 = x^2}$ $y\sqrt[3]{1-x} = x$

$y = x^2 + yx^3$ $y^3(1-x) = x^3$

y^3

$t^3/xt^2 = 1/1-x$

$t/x = 1/1-x$

$t = x/1-x$ $y = xt$

$y = x^2/1-x$

$y - yx = x^2.$

Examiner commentary

Quite a few candidates lost a mark here like this candidate, as they did not explicitly state that their working showed that the two sides are equal.

Exemplar 2 0 marks

8(ii)

$$\left(\frac{t}{1+t^3}\right)^3 + \left(\frac{t^2}{1+t^3}\right)^3 = \frac{t^3}{(1+t^3)^3} + \frac{t^6}{(1+t^3)^3}$$

$$= \frac{t^6+t^3}{(1+t^3)^3} \text{ M1 } = \frac{t^3(t^3+1)}{(1+t^3)^3} \text{ M1 } = \frac{t^3}{(1+t^3)^2} = xy$$

Because $xy = \left(\frac{t}{1+t^3}\right)\left(\frac{t^2}{1+t^3}\right) = \frac{t^3}{(1+t^3)^2}$ A0

(answer space continued on next page)

Examiner commentary

The other common error was to try and prove the result leading to problems with the algebraic manipulation and scored few or, like this candidate, no marks. The specification does define a number of assessment command words such as 'prove', 'determine' and 'show that', but the command word 'verify' comes under the heading of other command words, having their ordinary English meaning, i.e. to demonstrate that something is true, accurate or justified.

Question 9 (i)

9 The function $\mathrm{f}(x) = \dfrac{\mathrm{e}^x}{1-\mathrm{e}^x}$ is defined on the domain $x \in \mathbb{R},\ x \neq 0$.

(i) Find $\mathrm{f}^{-1}(x)$. **[3]**

Exemplar 1 — 3 marks

9(i)

$\cancel{f(x) = \frac{e^x}{1-e^x}}$ $\cancel{f(x)(1-e^x) = e^x}$

$\cancel{f(x) - e^x f(x) = e^x}$

$y = f(x)$ $y = \frac{e^x}{1-e^x}$ $y(1-e^x) = e^x$ $y - ye^x = e^x$

$y = e^x + ye^x$ $y = e^x(1+y)$ $e^x = \frac{y}{1+y}$

$x = \ln \frac{y}{1+y}$

renaming variables

$f^{-1}(x) = \ln \frac{x}{1+x}$

Examiner commentary

Many candidates struggled to finish off the equation often forgetting to swap *x* and *y* or using incorrect notation for $f^{-1}(x)$. This candidate's approach helped ensure full credit with a clear logical argument.

Question 9 (ii)

9 (ii) Write down the range of $f^{-1}(x)$. **[1]**

Exemplar 1 0 marks

9(ii)	
	range = $x \in \mathbb{R}$, $x \neq 0$

Examiner commentary

The *range* of a function should be given as a set of values of f(x) or in this question $f^{-1}(x)$. Examiners would condone the use of y rather than $f^{-1}(x)$ but here where the candidate gives a set of values of x the mark is not earned.

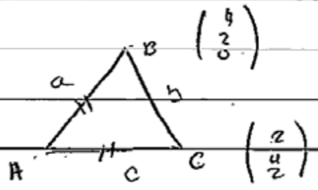
Question 10 (i)

10 Point A has position vector $\begin{pmatrix} a \\ b \\ 0 \end{pmatrix}$ where a and b can vary, point B has position vector $\begin{pmatrix} 4 \\ 2 \\ 0 \end{pmatrix}$ and point C has position vector $\begin{pmatrix} 2 \\ 4 \\ 2 \end{pmatrix}$. ABC is an isosceles triangle with AC = AB.

(i) Show that $a - b + 1 = 0$. **[4]**

Exemplar 1 — 2 marks

10(i)



$\vec{AB} = \vec{AC}$

$\begin{pmatrix} 4-a \\ 2-b \\ 0-0 \end{pmatrix} = \begin{pmatrix} 2-a \\ 4-b \\ 2-0 \end{pmatrix}$ M1

$\sqrt{(4-a)^2 + (2-b)^2} = \sqrt{(2-a)^2 + (4-b)^2 + 2^2}$ M1

$(4-a)^2 + (2-b)^2 = (2-a)^2 + (4-b)^2 + 4$

$a = 2,\ b = 3 \rightarrow \underline{2 - 3 + 1 = 0}$

$a - b = c$

$\vec{AB} - \vec{BC} = \vec{AC}$

$\vec{AB} - \vec{BC} = \vec{AB}$

$a - b = a \ (c = a)$

$b = 0;\ -a = 1$

$1 - 0 + 1 =$

$c + b = a$

$\vec{BC} = -\vec{AB} + \vec{AC}$

$\begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} = -\vec{AB} + \vec{AC}$

$\begin{pmatrix} -2 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} a-4 \\ b-2 \\ 0 \end{pmatrix} + \begin{pmatrix} 2-a \\ 4-b \\ 2 \end{pmatrix}$

$a - 4 + 2 - a = -2$

$b - 2 + 4 - b = 2$

$\vec{AC} + \vec{BC} = -\vec{AB}$

$c + b = -a$

$a - b + b = -a$

$a = -a$

$c + b = -a + b$

$c + a = 0 \rightarrow c = -a$

$a - b = c$

$a - b = -a$

$2a = b;\ a = 1/2 b$

$1/2 b - b + 1$

$= -1/2 b + 1 =$

Examiner commentary

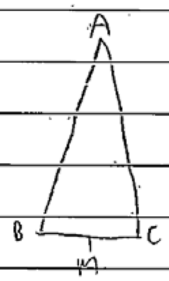
This candidate understood the relevant A Level maths to apply to this question in terms of finding vectors and their moduli. However they then did not expand and simplify to 'show that' the equation they ended up matched the given answer, with which could easily have led to this candidate earning full marks.

Question 10 (ii)

10 **(ii)** Determine the position vector of A such that triangle ABC has minimum area.

Exemplar 1 4 marks

10(ii)



$M: \begin{pmatrix} 3 \\ 3 \\ 1 \end{pmatrix}$ B1

$|BC| = 2\sqrt{3}$

area $= \frac{1}{2} \times |AM| \times |BC|$

Area is W $W = \sqrt{3}\,.\,|AM|$

$|AM| = \sqrt{(a-3)^2 + (b-3)^2 + 1^2} = \sqrt{a^2 - 6a + 9 + b^2 - 6b + 9 + 1} = \sqrt{a^2 + b^2 - 6a - 6b + 19}$

As $a - b + 1 = 0$, $a = b - 1$

$\cancel{W = \sqrt{3}\,(a^2}$

$W = \sqrt{3}\,((b-1)^2 + b^2 - 6(b-1) - 6b + 19)$ M1 M1

$= \sqrt{3}\,(b^2 - 2b + 1 + b^2 - 6b + b - 6b + 19)$

$W = \sqrt{3}\,(2b^2 - 13b + 20)$

$W = 2\sqrt{3}\,b^2 - 13\sqrt{3}\,b + 20\sqrt{3}$ M1

$\frac{dW}{db} = 4\sqrt{3}\,b - 13\sqrt{3}$ (Linear so only 1 stationary value)

@ stationary, $\frac{dW}{db} = 0$ $4\sqrt{3}\,b = 13\sqrt{3}$ $4b = 13$ $b = \frac{13}{4} = 3.25$

so $a = 2.25$

(assuming its minimum because linear)

$\begin{pmatrix} 2.25 \\ 3.25 \\ 0 \end{pmatrix}$ is A

Examiner commentary

This candidate has made an excellent attempt, unfortunately dropping the final two accuracy marks after misreading their own writing $-6(b-1)$ to give $-6b + b$ rather than the correct $-6b + 6$.

Question 11 (i)

11 Line 8 states that $\frac{a+b}{2} \geqslant \sqrt{ab}$ for a, $b \geqslant 0$. Explain why the result cannot be extended to apply in each of the following cases.

(i) One of the numbers a and b is positive and the other is negative. **[1]**

Exemplar 1 — 1 mark

11(i) You ~~ba~~ will ~~addd~~ end up with ab ~~a+b~~ as a negative number and you can't take the square root of a negative number

Examiner commentary

Even though this candidate has struggled throughout the paper, good examination technique has ensured that all accessible marks have been gained.

Question 11 (ii)

11 (ii) Both numbers a and b are negative. **[1]**

Exemplar 1 1 mark

11(ii)	Then $\frac{a+b}{2}$ would be negative but $\sqrt{ab}$ would be positive and $\frac{a+b}{2}$ would not be greater than or equal to $\sqrt{ab}$.

Examiner commentary

Some responses to question 11 parts (i) and (ii) were a bit convoluted. This response is clear and concise.

Question 12

12 Lines 5 and 6 outline the stages in a proof that $\frac{a+b}{2} \geqslant \sqrt{ab}$. Starting from $(a-b)^2 \geqslant 0$, give a detailed proof of the inequality of arithmetic and geometric means. **[3]**

Exemplar 1 — 3 marks

12

$(a-b)^2 \geq 0$

$a^2 - 2ab + b^2 \geq 0$

$a^2 + b^2 \geq 2ab$

$\cancel{\frac{a^2+b^2}{2} \geq ab}$

$\cancel{\sqrt{\frac{a^2+b^2}{2}} \geq \sqrt{ab}}$

$a^2 + b^2 + 2ab \geq 4ab$

$(a+b)^2 \geq 4ab$

$a+b \geq \sqrt{4ab}$

$a+b \geq 2\sqrt{ab}$

$\frac{a+b}{2} \geq \sqrt{ab}$

Examiner commentary

This candidate planned out their proof and therefore produced a clear and concise argument.

Exemplar 2 1 mark

12

$\cancel{(a^2 \geq b^2}$ $(a-b)^2 = a^2 + b^2 - 2ab$ B1

If $\cancel{a^2+b^2}$ $(a+b)^2 \geq 0$ then $a^2+b^2 \geq 2ab$

If we square root both sides after rearranging

$\dfrac{\sqrt{a^2+b^2}}{2^2} \geq \sqrt{ab}$

which

$\sqrt{a^2+b^2+2ab} = (a+b)$

So we really needed

$a^2+b^2 \geq 4ab$

which shows

$a+b/\sqrt{4} \geq \sqrt{ab}$

So

$a+b/2 \geq \sqrt{ab}.$

Examiner commentary

Candidates should be expecting proof questions as a part of Assessment Objective 2 that makes up 25% of the A Level assessment. If a candidate has not planned their way through an argument making sure they are clear on all the steps then, like this candidate, they soon collapse into waffle.

Question 13

13 Consider a geometric sequence in which all the terms are positive real numbers. Show that, for any three consecutive terms of this sequence, the middle one is the geometric mean of the other two. **[3]**

Exemplar 1 — 2 marks

13

Eg All geometric sequences can be written as
ar^n

Any three terms:
ar^n, ar^{n+1}, ar^{n+2} B1

geometric mean of two numbers $= \sqrt{ab}$
$a = ar^n$
$b = ar^{n+2}$
$a \times b = a^2r^{2n+2}$ M1
$\sqrt{a^2r^{2n+2}} = ar^{n+1}$ because all the indices have been divided by 2.

Examiner commentary

As in 8(ii) this is another example of the candidate losing the final mark for not 'completing' their argument. All that was needed was a sentence saying 'This shows that the geometric mean of the first and last is equal to the middle term'.

Question 14 (i)

14 **(i)** In Fig. C1.3, angle CBD = θ. Show that angle CDA is also θ, as given in line 23. **[2]**

Exemplar 1 1 mark

14(i)

$\angle CDB = \cancel{180}\ 90 - \theta$.

Angle in a ~~a~~ semicircle = 90° M1

so $ADB = 90°$

so ~~ADC~~ $CDA = ADB - CDB$

$CDA = 90 - (90 - \theta)$

$\angle CDA = \theta$.

Examiner commentary

The demand here was 'Show that …' meaning that steps needed to be given and reasons for their working stated. This candidate earns 1 mark for telling us that they have used 'angle in a semicircle = 90°'. However they do not say that they have used 'angles in a triangle = 90°' so they are limited to the first mark only.

Question 14 (ii)

14 **(ii)** Prove that $h = \sqrt{ab}$, as given in line 24. **[2]**

Exemplar 1 1 mark

14(ii)

M1

$\tan\theta = \frac{h}{a}$ $\tan\theta = \frac{h}{b}$

$\frac{h}{a} = \frac{h}{b}$

$\cancel{bh - ha = 0}$

$\cancel{h(b-a) =}$

$\tan^2\theta = \frac{h^2}{a^2}$ tan

Examiner commentary

The majority of candidates gained full marks here, the most common error, as in this example, was to switch the sides in the tangent expression.

Question 15

15 It is given in lines 31–32 that the square has the smallest perimeter of all rectangles with the same area. Using this fact, prove by contradiction that among rectangles of a given perimeter, $4L$, the square with side L has the largest area. **[3]**

Exemplar 1 3 marks

15 Let us suppose, for a contradiction, that there is another rectangle with side lengths a and b which has a larger area than the square (area of L^2) and a perimeter of $4L$. M1

$a+b=2L$

$\frac{2L}{2} \geq \sqrt{ab}$

$L^2 \geq ab$

so the largest area such a rectangle could have would be L^2. A1

But the square has the smallest perimeter of all rectangles with area L^2.

So the rectangle can't have area L^2 and perimeter $4L$ without it being a square.

So by a contradiction it is proved. A1

M1

Examiner commentary

Very few candidates appreciated how to construct a proof by contradiction. To get any marks, examiners were looking for an initial statement that could be followed through to a contradiction. This candidate has set out a clear initial statement, followed by a mathematical argument to demonstrate a contradiction. A classroom review of the proofs that $\sqrt{2}$ and $\sqrt{3}$ are irrational would be a useful starting point for this topic.

Some candidates ignored the question and attempted to prove by deduction. Some of them completed their prove saying 'By deduction this shows ….'

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Qualification and notional component raw mark grade boundaries June 2018 series

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New AS Levels

AS GCE Mathematics B (MEI)										
				Max Mark	a	b	c	d	e	u
H630	01	Pure Mathematics and Mechanics	Raw	70	44	38	33	28	23	0
H630	02	Pure Mathematics and Statistics	Raw	70	50	45	39	33	28	0
			Overall	140	94	83	72	61	51	0

AS GCE Further Mathematics B (MEI) (H635)										
				Max Mark	a	b	c	d	e	u
Y410	01	Core Pure	Raw	60	46	41	36	32	28	0
Y411	01	Mechanics a	Raw	60	37	32	27	22	18	0
Y412	01	Statistics a	Raw	60	42	38	34	30	26	0
Y413	01	Modelling with Algorithms	Raw	60	37	33	29	25	22	0
Y414	01	Numerical Methods	Raw	60	35	29	24	19	14	0
Y415	01	Mechanics b	Raw		No entry in June 2018					
Y416	01	Statistics b	Raw	60	43	38	33	28	24	0
H635		*Option Y410+Y411+Y412*	Overall	180	125	111	98	85	72	0
H635		*Option Y410+Y411+Y413*	Overall	180	120	107	94	81	68	0
H635		*Option Y410+Y411+Y414*	Overall	180	118	103	88	74	60	0
H635		*Option Y410+Y412+Y413*	Overall	180	125	112	100	88	76	0
H635		*Option Y410+Y412+Y414*	Overall	180	123	109	95	81	68	0
H635		*Option Y410+Y412+Y416*	Overall	180	131	117	104	91	78	0
H635		*Option Y410+Y413+Y414*	Overall	180	118	104	90	77	64	0

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New A Levels

A Level Mathematics B (MEI)												
				Max Mark	**a***	**a**	**b**	**c**	**d**	**e**	**u**
H640	01	Pure Mathematics and Mechanics	Raw	100	81	74	67	59	52	45	0
H640	02	Pure Mathematics and Statistics	Raw	100	75	68	61	54	47	40	0
H640	03	Pure Mathematics and Comprehension	Raw	75	62	55	48	42	36	30	0
			Overall	275	218	197	176	155	135	115	0