OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics and Statistics
Question Paper

Wednesday 13 June 2018 – Morning

Time allowed: 2 hours

7020885537

You must have:
• Printed Answer Booklet

You may use:
• a scientific or graphical calculator

INSTRUCTIONS

- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION

- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **16** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \frac{a(1-r^n)}{1-r}$

$S_\infty = \frac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^nC_1 a^{n-1}b + {}^nC_2 a^{n-2}b^2 + \ldots + {}^nC_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^nC_r = {}_nC_r = \binom{n}{r} = \frac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots + \frac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\operatorname{cosec}^2 x$
$\operatorname{cosec} x$	$-\operatorname{cosec} x \cot x$

Quotient Rule $y = \frac{u}{v},\ \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{v\frac{\mathrm{d}u}{\mathrm{d}x} - u\frac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim_{h \to 0} \frac{\mathrm{f}(x+h) - \mathrm{f}(x)}{h}$

Integration

$\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A\sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A\sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B} \qquad \left(A \pm B \neq (k+\frac{1}{2})\pi\right)$

Numerical methods

Trapezium rule: $\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A)+\mathrm{P}(B)-\mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B\,|\,A) = \mathrm{P}(B)\mathrm{P}(A\,|\,B)$ **or** $\mathrm{P}(A\,|\,B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X=r) = {}^n\mathrm{C}_r\,p^r q^{n-r}$ where $q = 1-p$

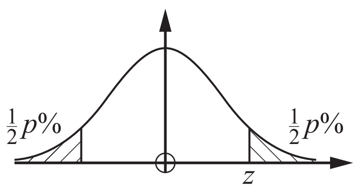
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu,\sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u+at$

$s = ut+\frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2+2as$

$s = vt-\frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u}+\mathbf{a}t$

$\mathbf{s} = \mathbf{u}t+\frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t-\frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (21 marks)

1 Show that $\sqrt{27}+\sqrt{192}=a\sqrt{b}$, where a and b are prime numbers to be determined. **[2]**

2 Solve the inequality $|2x+1|<5$. **[3]**

3 The probability that Chipping FC win a league football match is P(W) = 0.4.

(i) Calculate the probability that Chipping FC fail to win each of their next two league football matches. **[1]**

The probability that Chipping FC lose a league football match is P(L) = 0.3.

(ii) Explain why $\mathrm{P}(W)+\mathrm{P}(L)\neq 1$. **[1]**

4 A survey of the number of cars per household in a certain village generated the data in Fig. 4.

Number of cars	0	1	2	3	4
Number of households	8	22	31	27	7

Fig. 4

(i) Calculate the mean number of cars per household. **[1]**

(ii) Calculate the standard deviation of the number of cars per household. **[1]**

5 **(i)** (A) Sketch the graph of $y=3^x$. **[1]**

(B) Give the coordinates of any intercepts. **[1]**

The curve $y=\mathrm{f}(x)$ is the reflection of the curve $y=3^x$ in the line $y=x$.

(ii) Find f(x). **[1]**

6 **(i)** Express $7\cos x-24\sin x$ in the form $R\cos(x+\alpha)$, where $0<\alpha<\frac{\pi}{2}$. **[3]**

(ii) Write down the range of the function

$$\mathrm{f}(x)=12+7\cos x-24\sin x\,,\qquad 0\leqslant x\leqslant 2\pi.$$

[2]

7 Find $\int\left(4\sqrt{x}-\frac{6}{x^3}\right)\mathrm{d}x$. **[4]**

Answer **all** the questions

Section B (79 marks)

8 Every morning before breakfast Laura and Mike play a game of chess. The probability that Laura wins is 0.7. The outcome of any particular game is independent of the outcome of other games. Calculate the probability that, in the next 20 games,

(i) Laura wins exactly 14 games, **[2]**

(ii) Laura wins at least 14 games. **[2]**

9 At the end of each school term at North End College all the science classes in year 10 are given a test. The marks out of 100 achieved by members of set 1 are shown in Fig. 9.

3	5
4	0 9
5	2 3 6
6	0 1 3 5 6
7	0 1 2 5 6 8 9 9
8	3 4 6 6 8 8 9
9	5 5 5 6 7

Key 5 | 2 represents a mark of 52

Fig. 9

(i) Describe the shape of the distribution. **[1]**

(ii) The teacher for set 1 claimed that a typical student in his class achieved a mark of 95.
How did he justify this statement? **[1]**

(iii) Another teacher said that the average mark in set 1 is 76. How did she justify this statement? **[1]**

Benson's mark in the test is 35. If the mark achieved by any student is an outlier in the lower tail of the distribution, the student is moved down to set 2.

(iv) Determine whether Benson is moved down to set 2. **[2]**

10 The screenshot in Fig. 10 shows the probability distribution for the continuous random variable X, where $X \sim \mathrm{N}(\mu, \sigma^2)$.

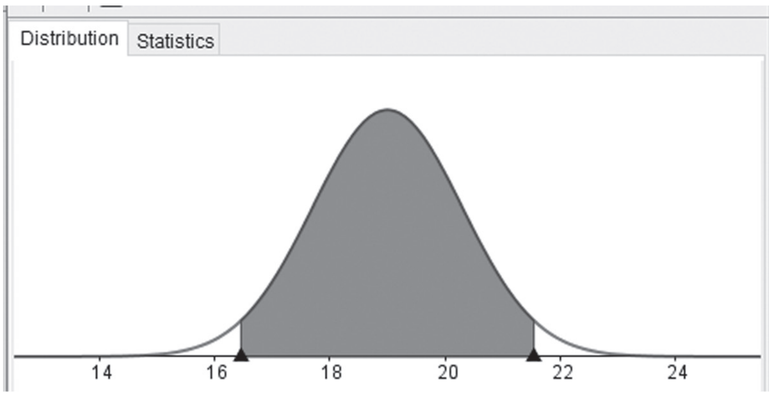


Fig. 10

The area of each of the unshaded regions under the curve is 0.025. The lower boundary of the shaded region is at 16.452 and the upper boundary of the shaded region is at 21.548.

(i) Calculate the value of μ. **[1]**

(ii) Calculate the value of σ^2. **[3]**

(iii) Y is the random variable given by $Y = 4X + 5$.

(A) Write down the distribution of Y. **[3]**

(B) Find $\mathrm{P}(Y > 90)$. **[1]**

11 The discrete random variable X takes the values 0, 1, 2, 3, 4 and 5 with probabilities given by the formula

$$\mathrm{P}(X = x) = k(x + 1)(6 - x) .$$

(i) Find the value of k. **[2]**

In one half-term Ben attends school on 40 days. The probability distribution above is used to model X, the number of lessons per day in which Ben receives a gold star for excellent work.

(ii) Find the probability that Ben receives no gold stars on each of the first 3 days of the half-term and two gold stars on each of the next 2 days. **[2]**

(iii) Find the expected number of days in the half-term on which Ben receives no gold stars. **[2]**

12 You must show detailed reasoning in this question.

In the summer of 2017 in England a large number of candidates sat GCSE examinations in **both** mathematics **and** English. 56% of these candidates achieved at least level 4 in mathematics and 80% of these candidates achieved at least level 4 in English. 14% of these candidates did not achieve at least level 4 in either mathematics or English.

Determine whether achieving level 4 or above in English and achieving level 4 or above in mathematics were independent events. **[5]**

13 Each weekday Keira drives to work with her son Kaito. She always sets off at 8.00 a.m. She models her journey time, x minutes, by the distribution $X \sim \mathrm{N}(15, 4)$.

Over a long period of time she notes that her journey takes less than 14 minutes on 7% of the journeys, and takes more than 18 minutes on 31% of the journeys.

(i) Investigate whether Keira's model is a good fit for the data. **[3]**

Kaito believes that Keira's value for the variance is correct, but realises that the mean is not correct.

(ii) Find, correct to two significant figures, the value of the mean that Keira should use in a refined model which does fit the data. **[2]**

Keira buys a new car. After driving to work in it each day for several weeks, she randomly selects the journey times for n of these days. Her mean journey time for these n days is 16 minutes. Using the refined model she conducts a hypothesis test to see if her mean journey time has changed, and finds that the result is significant at the 5% level.

(iii) Determine the smallest possible value of n. **[5]**

14 The pre-release material includes data on unemployment rates in different countries. A sample from this material has been taken. All the countries in the sample are in Europe. The data have been grouped and are shown in Fig 14.1.

Unemployment rate	0–	5–	10–	15–	20–	35–50
Frequency	15	21	5	5	2	2

Fig. 14.1

A cumulative frequency curve has been generated for the sample data using a spreadsheet. This is shown in Fig. 14.2.

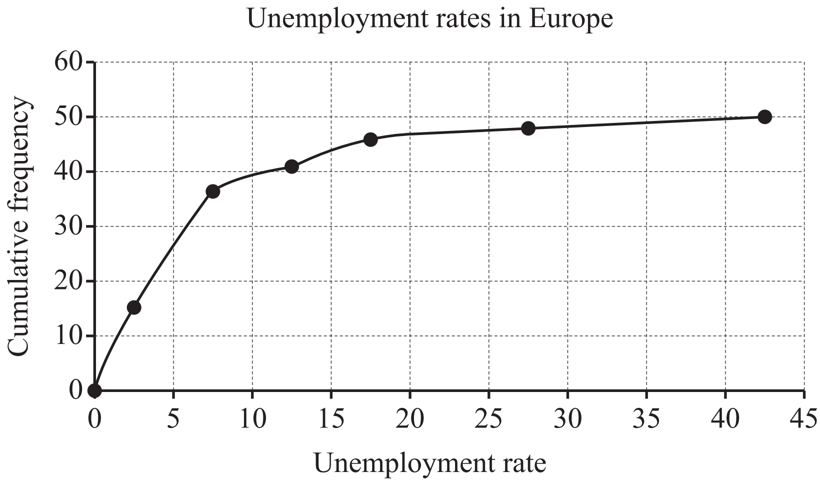


Fig. 14.2

Hodge used Fig. 14.2 to estimate the median unemployment rate in Europe. He obtained the answer 5.0. The correct value for this sample is 6.9.

(i) (*A*) There is a systematic error in the diagram.

- Identify this error.
- State how this error affects Hodge's estimate. **[2]**

(*B*) There is another factor which has affected Hodge's estimate.

- Identify this factor.
- State how this factor affects Hodge's estimate. **[2]**

(ii) Use your knowledge of the pre-release material to give another reason why any estimation of the median unemployment rate in Europe may be unreliable. **[1]**

(iii) Use your knowledge of the pre-release material to explain why it is very unlikely that the sample has been randomly selected from the pre-release material. **[1]**

The scatter diagram shown in Fig. 14.3 shows the unemployment rate and life expectancy at birth for the 47 countries in the sample for which this information is available.

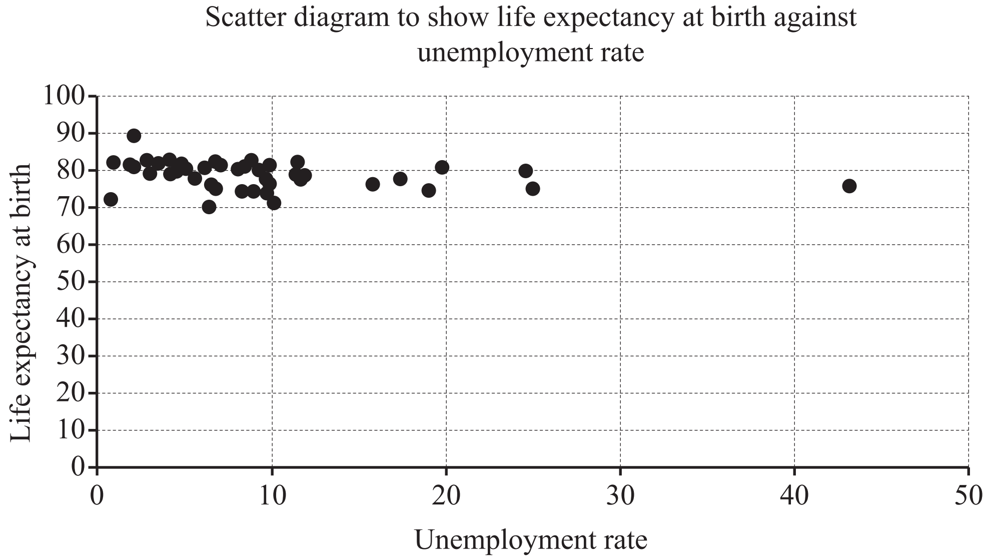


Fig. 14.3

The product moment correlation coefficient for the 47 items in the sample is – 0.2607.
The p-value associated with $r = -0.2607$ and $n = 47$ is 0.0383.

(iv) Does this information suggest that there is an association between unemployment rate and life expectancy at birth in countries in Europe? **[2]**

Hodge uses the spreadsheet tools to obtain the equation of a line of best fit for this data.

(v) The unemployment rate in Kosovo is 35.3, but there is no data available on life expectancy. Is it reasonable to use Hodge's line of best fit to estimate life expectancy at birth in Kosovo? **[1]**

15 **You must show detailed reasoning in this question.**

The equation of a curve is

$$y^3 - xy + 4\sqrt{x} = 4 \ .$$

Find the gradient of the curve at each of the points where $y = 1$. **[9]**

16 In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for t hours. Over the course of the year she realises that her percentage mark for a test, p, may be modelled by the following formula, where A, B and C are constants.

$$p = A - B(t - C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.

- Aaishah had a mark of 22 when she didn't spend any time revising.

(i) Find the values of A, B and C. **[3]**

(ii) According to the model, if Aaishah revises for 45 minutes on the night before the test, what mark will she achieve? **[2]**

(iii) What is the maximum amount of time that Aaishah could have spent revising for the model to work? **[2]**

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p.

$$p = 22 + 68(1 - e^{-0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
p	59	84	89

Fig. 16

(iv) Verify that the data is consistent with the new formula. **[2]**

(v) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future. Determine whether or not this is good advice. **[2]**

17 **(i)** Express $\dfrac{(x^2 - 8x + 9)}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(ii) Express y in terms of x given that

$$\frac{dy}{dx} = \frac{y(x^2 - 8x + 9)}{(x+1)(x-2)^2} \text{ and } y = 16 \text{ when } x = 3.$$

[7]

END OF QUESTION PAPER

OCR
Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/02 Pure Mathematics with Statistics

Printed Answer Booklet

Wednesday 13 June 2018 – Morning

Time allowed: 2 hours

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- Question Paper H640/02 (inserted)

You may use:
- a scientific or graphical calculator

First name										
Last name										
Centre number						Candidate number				

INSTRUCTIONS
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1	
2	
3(i)	
3(ii)	

4(i)	
4(ii)	
5(i)(*A*)	
5(i)(*B*)	
5(ii)	

6(i)	
6(ii)	
7	

8(i)	
8(ii)	
9(i)	
9(ii)	

9(iii)	
9(iv)	
10(i)	

10(ii)	
10(iii)(*A*)	
10(iii)(*B*)	

11(i)	
11(ii)	
11(iii)	

12	
13(i)	

13(ii)	
13(iii)	

14(i)(*A*)

14(i)(*B*)

14(ii)

14(iii)

14(iv)	
14(v)	
15	
	(continued on next page)

15	(continued)

16(i)	
16(ii)	
16(iii)	

16(iv)	
16(v)	
17(i)	

17(ii)

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GCE

Mathematics B (MEI)

Unit **H640/02**: Pure Mathematics and Statistics

Advanced Subsidiary GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	**Meaning**
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep*' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

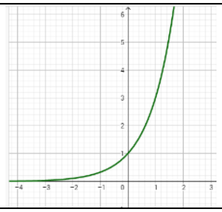
f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. 'Fresh starts' will not affect an earlier decision about a misread. Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs	Guidance	
1			$3\sqrt{3}$ or $8\sqrt{3}$ seen $[3\sqrt{3}+8\sqrt{3}]=11\sqrt{3}$	**M1** **A1** **[2]**	**1.1** **2.1**		
2			$-5 < 2x+1 < 5$ $-6 < 2x < 4$ $-3 < x < 2$	**M1** **A1** **A1** **[3]**	**2.1** **1.1** **1.1**	$-(2x+1)<5$ oe and $2x+1<5$ -3 and 2 identified $-3<x<2$ allow $x > -3$ **and** $x < 2$	or $(2x+1)^2 < 25$ if **M0** allow **B1** for either condition identified
3	**(i)**		0.36	**B1** **[1]**	**1.1**		
3	**(ii)**		P(draw) $\neq$ 0 oe	**B1** **[1]**	**2.4**	allow any comment which identifies that other outcomes are possible	eg winning and losing are not exhaustive
4	**(i)**		2.031578947….rounded to two or more sf isw BC	**B1** **[1]**	**1.1**	NB 2.0, 2.03 or 2.032	
4	**(ii)**		1.076367330…rounded to two or more sf isw BC	**B1** **[1]**	**1.1**	NB 1.1, 1.08 or 1.076	
5	**(i)**	*A*		**B1** **[1]**	**1.2**	correct shape in both quadrants	condone touching the *x*-axis, but not cutting it
5	**(i)**	*B*	(0, 1)	**B1** **[1]**	**1.1**	do not allow just $y = 1$	

Question			Answer	Marks	AOs	Guidance	
5	(ii)		$(\mathrm{f}(x)=)\log_3 x$	**B1** **[1]**	1.1	allow eg $\frac{\log x}{\log 3}$	
6	(i)		$R = 25$ $\tan^{-1}\left(\frac{24}{7}\right)$ or $\sin^{-1}\left(\frac{24}{25}\right)$ or $\cos^{-1}\left(\frac{7}{25}\right)$ $25\cos(x + 1.29)$	**B1** **M1** **A1** **[3]**	1.1 1.1 1.1	 α = 1.28700221759 rounded to 2 or more sf	 73.739795° rounded to 2 or more sf may imply **M1A0** allow **A1** for α found to 2 or more sf
6	(ii)		$12 \pm$ *their* 25 $-13 \le \mathrm{f}(x) \le 37$	**M1** **A1** **[2]**	3.1a 1.1	or one of – 13 and 37 identified allow eg from – 13 to 37 inclusive	 **A0** if inequality is strict
7			$kx^{\frac{3}{2}}$ kx^{-2} $\frac{8}{3}x^{\frac{3}{2}}$ or $+ 3x^{-2}$ seen $\frac{8}{3}x^{\frac{3}{2}} + 3x^{-2} + c$ isw	**M1** **M1** **A1** **A1** **[4]**	1.1 1.1 1.1 1.1		
8	(i)		use of $B \sim (20, 0.7)$ soi 0.191638982753…rounded to 2 or more dp isw BC	**M1** **A1** **[2]**	3.1b 1.1	 NB 0.1916 or 0.192 or 0.19	

Question			Answer	Marks	AOs	Guidance	
8	(ii)		$P(X \le 13)$ found soi 0.608009811813…rounded to 2 or more dp isw BC	M1 A1 [2]	3.1b 1.1	NB 0.391990188187 NB 0.6080 or 0.608 or 0.61	**M0** if $P(X = 13)$ used NB 0.1643… if **M0** allow **SC1** for $1 - P(X \le 14) = 1 - 0.58362..$ $=0.41637083$ rounded to 2 or more dp
9	(i)		negative skew	B1 [1]	1.2		
9	(ii)		(used) the mode	B1 [1]	1.1		
9	(iii)		(used) the median	B1 [1]	1.1		
9	(iv)		$61 - 1.5 \times (88 - 61)$ $20.5 < 35$ [so 35 is not an outlier] so he does not move to set 2	M1 A1 [2]	2.1 2.2b	*Alternatively,* $73.61 - 2 \times 17.03$ $39.6 > 35$ [so 35 is an outlier] so he moves to set 2	allow eg only marks below 20.5 (or 39.6) would lead to a move down plus correct conclusion
10	(i)		[μ =]19	B1 [1]	1.1		

Question			Answer	Marks	AOs	Guidance	
10	(ii)		$1.96 = \frac{21.548 - 19}{\sigma}$	M1	3.1a	or $-1.96 = \frac{16.452 - 19}{\sigma}$	NB 1.959963985…rounded to 3 or more sf **M0** if $z = 2$
			[σ =] awrt 1.3	A1	1.1	may be implied by final answer	
			[σ^2 =] awrt 1.69	A1 [3]	1.1	allow **B3** for awrt 1.69 unsupported	
10	(iii)	*A*	[μ =] 4 × *their* 19 + 5	M1	2.1		
			[σ^2 =] 4^2 × *their* 1.69 or σ = 4× *their* 1.3	M1	1.1		
			$[Y \sim]\,N(81, 5.2^2)$ oe	A1 [3]	1.1	NB 27.04	
10	(iii)	*B*	0.04175 or 0.0417 or 0.042 BC	B1 [1]	1.1		NB 0.0417462427103
11	(i)		$k(1\times6+2\times5+3\times4+4\times3+5\times2+6\times1)=1$ oe	M1	3.1a	allow one slip in arithmetic	
			$[k =]\frac{1}{56}$ isw	A1 [2]	1.1	**B2** if unsupported	
11	(ii)		$(6\times k)^3 \times (12\times k)^2$ oe seen	M1	2.1	FT their k	
			$\frac{243}{4302592}$ or 0.000056477584 rounded to 2 or more sf	A1 [2]	1.1		
11	(iii)		40 × 6k	M1	3.1b	FT their k	
			4.286 or 4.29 or 4.3	A1 [2]	3.2b	mark the final answer	

Question			Answer	Marks	AOs	Guidance	
12			use of contingency table or Venn diagram or P(*A* or *B*) = P(*A*) + P(*B*) – P(*A* and *B*)	M1	3.1b	0.56, 0.8 and 0.14 must be correctly placed; eg 1 – 0.14 = 0.56 + 0.8 – P(*A* and *B*)	where *A* denotes "passing" maths and *B* denotes "passing" English
			P(*A* and *B*) = 0.5	A1	2.1	or P(*A*/*B*) = $\frac{0.5}{0.80}$	the first **M1A1** may be awarded for working with percentages
			P(*A*) × P(*B*) = 0.56 × 0.80	M1	1.1		
			= 0.448 seen	A1	1.1	= 0.625 or ⅝ seen	allow equivalent argument based on showing *A*' and *B*' not independent
			$0.448 \neq 0.5$ or $0.56 \times 0.80 \neq 0.5$ so not independent	A1 [5]	3.2a	0.625 or ⅝ $\neq 0.56$	
13	(i)		calculation of P(*X* < 14) and P(*X* > 18)	M1	3.4	or solves $-1.476 = \frac{14-\mu}{\sigma}$ and $0.496 = \frac{18-\mu}{\sigma}$ simultaneously	or solves $-1.476 = \frac{x-15}{2}$ and $0.496 = \frac{x-15}{2}$
			0.3085 and 0.0668 to 1 sf or better	A1	1.1	$\mu \approx 17$ and $\sigma \approx 2.02$	*x* = 12.048 and 15.992 to nearest whole number or better
			these figures do not support the model	A1 [3]	3.5a	17 is (relatively) far from 15 so not a good fit the second **A1** is only available if the first **A1** is awarded	which are not close to 14 and 18
						allow **SC2** for showing the model is not a good fit for either value with all working correct or for a complete argument based on symmetry which refers to both tails	or $\frac{14-15}{2}$ and $\frac{18-15}{2}$ evaluated – 0.5 and 1.5 obtained which are not close to −1.476 and 0.496 respectively

Question			Answer	Marks	AOs	Guidance	
13	(ii)		$\Phi^{-1}(0.07) = -1.476 = \frac{14-\mu}{2}$ [μ = 16.95] **OR** $\Phi^{-1}(0.69) = 0.496 = \frac{18-\mu}{2}$ [μ = 17.008] [μ =]17	**M1** **A1** **[2]**	**3.5c** **2.4**	*alternatively* since the variance is assumed to be correct, the mean must be as far above the midpoint as it was previously below it. 16 + 1 = 17	if **M0** allow **B2** for 17 unsupported
13	(iii)		$z = \pm$ 1.96 used $\frac{16-\mu}{\frac{2}{\sqrt{n}}} < -1.96$ or $\frac{\mu-16}{\frac{2}{\sqrt{n}}} > 1.96$ $\sqrt{n}$ isolated from their $\frac{16-\mu}{\frac{\sigma}{\sqrt{n}}} < -1.96$ oe [n >] 15.3664 – 15.4 n = 16 cao	**B1** **M1** **M1** **A1** **A1** **[5]**	**1.1a** **3.1b** **2.1** **3.4** **2.2b**	allow method marks only if other z – value, eg – 1.645 used; FT μ eg $\sqrt{n} > 2 \times 1.96$ previous **A1** must be awarded for the award of final **A1**	NB 1.959963985…rounded to 3 or more sf **M0** if other value for σ used all marks are available if works with = instead of < or > throughout, but withhold final **A1** if works with < instead of > or > instead of < throughout
14	(i)	*A*	the cumulative frequencies have been plotted against the mid-points of the class intervals, mis-plotting [at centre of each class] reduces estimate (by 2.5) oe	**B1** **B1** **[2]**	**2.4** **2.4**		

Question			Answer	Marks	AOs	Guidance	
		B	grouped data has been used grouping has slightly reduced the error introduced by misplotting (because the error is less than 2.5)	**B1** **B1** **[2]**	**2.4** **2.4**	or eg Hodge used the graph (instead of the raw data)	
14	**(ii)**		percentage unemployment is often estimated oe	**E1** **[1]**	**2.4**	allow data (on percentage unemployment) is not available for all countries **in Europe** oe	
14	**(iii)**		there are many other countries in the pre-release material; it is very unlikely that a random sample would only include European countries.	**E1** **[1]**	**2.4**		
14	**(iv)**		negative correlation / association (may be embedded) comparison of *p*-value with 0.05 or 0.01 or other appropriate significance level and supporting comment	**B1** **B1** **[2]**	**2.2b** **2.2b**	if **B0B0** allow **SC2** for eg comment on no significant association justified by comparison of *p*-value with appropriate significance level (eg 0.025)	
14	**(v)**		(even though this is interpolation), the scatter / weak correlation / presence of an outlier would suggest that the use of of a line of best fit is inappropriate	**E1** **[1]**	**2.2b**	allow explanation based on the value for Kosovo being an outlier or on it lying in the (large) gap in the scatter	

Question			Answer	Marks	AOs	Guidance	
15			substitution of $y = 1$	M1	1.1a		
			$x - 4\sqrt{x} + 3 = 0$ or $4\sqrt{x} = x + 3$	A1	2.1		
			$x = 1$ or 9	A1	1.1		
			$3y^2 \frac{dy}{dx}$	B1	3.1a		
			$-x \times \frac{dy}{dx} - y$ or $x \times \frac{dy}{dx} + y$	M1	2.1	allow one sign error	
			$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} - y + \frac{2}{\sqrt{x}} [= 0]$	A1	1.1		
			substitution of $y = 1$ and *their* $x = 1$ or *their* $x = 9$	M1	1.1	dependent on at least two terms correct on LHS following differentiation	allow following wrong rearrangement after differentiating
			$m = -\frac{1}{2}$ [at (1, 1)]	A1	1.1		
			$m = -\frac{1}{18}$ [at (1, 9)]	A1	1.1	allow – 0.05555…to 2 sf or better	
				[9]			
16	(i)		$C = 2$	B1	3.3	since max when $t = 2$	
			$A = 62$	B1	3.3	since max when $(t - 2)^2 = 0$	
			$B = 10$	B1	1.1	from substitution of 22, 62 and 2	
				[3]			
16	(ii)		substitution of 0.75 in $p = 62 - 10(t - 2)^2$	M1	3.4	FT *their* 2, 62, 10	
			46	A1	1.1	allow 46.375 rounded to 2 or more sf	
				[2]			

Question			Answer	Marks	AOs	Guidance	
16	(iii)		*their* $62 - 10(t-2)^2 = 0$	M1	3.4	or ≥ 0 or > 0 for **M1** NB $t = 2 + \sqrt{6.2}$	
			[t =] 4 hours 29 minutes or 4 hours 30 minutes	A1 [2]	2.4	allow 4.49 or 4.5 [hours]	
16	(iv)		substitution of t = 1, 3 and 5	M1	3.4		if **M0** allow **SC1** for two values correctly found and shown to be consistent or supporting comment made
			awrt 59.4 ≈ 59 awrt 83.8 ≈ 84 awrt 88.8 ≈ 89	A1 [2]	3.5a	or awrt 59.4, 83.8 and 88.8 found and supporting comment made eg they are approximately the same as the values in the table	
16	(v)		$p \to 90$ as $t \to$ large or when t = 12 p = 89.99539…rounded to 2 or more sf	B1	3.5a		
			comparison with value of p for t = 5 eg model predicts p = 89 for t = 5 and p = 90 for t = 12 so not good advice	B1 [2]	3.5a	or model predicts p = 90 for (any) $t \geq 7$ so not good advice	allow equivalent comment on 7 hours work for one extra mark
17	(i)		$\frac{A}{(x+1)} + \frac{B}{(x-2)} + \frac{C}{(x-2)^2}$	B1	3.1a	may be seen later	
			$x^2 - 8x + 9 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$	M1	2.1		
			$A = 2$ $B = -1$ $C = -1$	A1 A1 A1 [5]	1.1 1.1 1.1 1.1	$\frac{2}{(x+1)} - \frac{1}{(x-2)} - \frac{1}{(x-2)^2}$	

Question			Answer	Marks	AOs	Guidance	
17	(ii)		$\int \frac{dy}{y} = \int \frac{x^2 - 8x + 9}{(x+1)(x-2)^2}\,dx$ soi	M1*	3.1a	allow omission of integral signs and/or omission of d*y* and/or d*x*	
			use of their partial fractions in integration	M1*	2.1	allow one sign error and/or one coefficient error	condone use of brackets instead of modulus signs; these two **A** marks are only available following the award of **both M** marks
			$\ln\lvert y\rvert = 2\ln\lvert x+1\rvert - \ln\lvert x-2\rvert + \frac{1}{x-2} + c$	A1	1.1	**A1** for any correct natural log integral on RHS FT *their* $\frac{2}{x+1}$ or *their* $\frac{-1}{x-2}$	
				A1	1.1	**A1** for $\frac{1}{x-2}$ FT *their* $\frac{k}{(x-2)^2}$	
			substitution of $y = 16$ and $x = 3$	M1dep*	1.1	expression must include + c and must include at least one natural log term; may be awarded after exponentiating	may be awarded following collection of like terms, which may contain errors NB $c = -1$
			correctly exponentiate both sides of their equation	M1	1.1		
			$y = \frac{(x+1)^2}{x-2} e^{\frac{3-x}{x-2}}$ oe	A1 [7]	2.1	eg $\frac{(x+1)^2}{x-2} e^{\frac{1}{x-2}}\, e^{-1}$	

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Examiners' report

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/02 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H640/02 series overview

This was the second paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates. This paper contributes 36.4% of the total A-level and assesses content from pure mathematics and statistics.

Candidates are expected to have studied statistics using the large data set and to have routinely used spreadsheets, graphing software and calculators when studying this course.

To do well in this component, candidates need to be able to apply their knowledge of the syllabus content in a variety of modelling and statistical contexts, and to make efficient use of calculator technology.

Notation for sample variance and sample standard deviation

Sample variance

$$s^2 = \frac{1}{n-1}S_{xx} \text{ where } S_{xx} = \sum(x_i - \bar{x})^2 = \sum x_i^2 - \frac{\left(\sum x_i\right)^2}{n} = \sum x_i^2 - n\bar{x}^2$$

Standard deviation

$$s = \sqrt{\text{variance}}$$

The notations s^2 and s for sample variance and sample standard deviation, respectively, are written into both British Standards (BS3534-1, 2006) and International Standards (ISO 3534). In this specification the usage will be consistent with these definitions. Thus the meanings of 'sample variance', denoted by s^2, and 'sample standard deviation', denoted by s, are defined to be calculated with divisor (n-1).

Students should be aware of the variations in notation used by manufacturers on calculators and know what the symbols on their particular models represent

Section A overview

Section A questions are designed to give all candidates an opportunity to do some of the questions on the paper as they require little reading or interpretation. Most candidates did very well in section A. The content was clearly understood, and the work was clearly and efficiently presented.

Question 1

1 Show that $\sqrt{27}+\sqrt{192}=a\sqrt{b}$, where a and b are prime numbers to be determined. **[2]**

This is a show that question; simply obtaining the values for *a* and *b* from the calculator, without any written justification, would not gain full credit.

Question 2

2 Solve the inequality $|2x+1|<5$. **[3]**

This question proved routine for the majority of candidates.

Question 3 (i)

3 The probability that Chipping FC win a league football match is $\mathrm{P}(W)=0.4$.

(i) Calculate the probability that Chipping FC fail to win each of their next two league football matches. **[1]**

A small minority of candidates found the probability to win the next two league football matches.

Question 3 (ii)

The probability that Chipping FC lose a league football match is $\mathrm{P}(L)=0.3$.

(ii) Explain why $\mathrm{P}(W)+\mathrm{P}(L)\neq 1$. **[1]**

Generally answered well, although some mini essays seen.

Question 4 (i)

4 A survey of the number of cars per household in a certain village generated the data in Fig. 4.

Number of cars	0	1	2	3	4
Number of households	8	22	31	27	7

Fig. 4

(i) Calculate the mean number of cars per household. **[1]**

Whilst it is acceptable to calculate the mean using a formal written method, there is an expectation in the new specification that candidates will use the statistical functions on their calculators for parts (i) and (ii), hence the single mark allocation.

Question 4 (ii)

(ii) Calculate the standard deviation of the number of cars per household. **[1]**

Candidates who did well in this question made efficient use of the appropriate calculator function.

Candidates who did less well made laborious calculations and slipped up in the arithmetic.

Misconception The choice of appropriate formula (using *n*, *n*-1 or other denominator corrections) is beyond the scope of this A Level Maths qualification, and the use of a single formula for all contexts is expected. The H640 OCR B (MEI) specification and formulae sheet makes clear that the divisor (*n*-1) should be used

5 **(i)** (*A*) Sketch the graph of $y = 3^x$. **[1]**

Question 5 (i) (A)

Sketches should make clear that the function tends towards *y*=0 but does not cut the *x*-axis

Question 5 (i) (B)

(*B*) Give the coordinates of any intercepts. **[1]**

A clear indication of the coordinates (0,1) was required and not just *y*=1.

Question 5 (ii)

The curve $y = \mathrm{f}(x)$ is the reflection of the curve $y = 3^x$ in the line $y = x$.

(ii) Find $\mathrm{f}(x)$. **[1]**

There was an expectation that candidates would use base 3 logarithms, but a variety of correct functions were given and gained credit.

Question 6 (i)

6 (i) Express $7\cos x - 24\sin x$ in the form $R\cos(x+\alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **[3]**

The majority of candidates gained full credit, with careless arithmetic resulting in dropped accuracy marks on this routine item.

Question 6 (ii)

(ii) Write down the range of the function

$$f(x) = 12 + 7\cos x - 24\sin x, \quad 0 \leqslant x \leqslant 2\pi.$$

[2]

Some candidates answered their own question, taken directly from part (i).

Question 7

7 Find $\int \left(4\sqrt{x} - \frac{6}{x^3}\right) dx$. **[4]**

Generally answered well, but the + *c* was often missed.

Section B overview

Candidates who did well in this section were able to present clear arguments with supporting calculations. They were familiar with the appropriate technology and were able to critique statistical diagrams.

Candidates who did less well did not show full details of their working. In some cases, it appeared that the demand of the question had not been properly understood, and some candidates did not appear to be familiar with all the functionality of their calculator.

Question 8 (i)

8 Every morning before breakfast Laura and Mike play a game of chess. The probability that Laura wins is 0.7. The outcome of any particular game is independent of the outcome of other games. Calculate the probability that, in the next 20 games,

(i) Laura wins exactly 14 games, **[2]**

This was answered well when clear working was shown. Mistakes may have been due to going straight to the calculator and mistakenly finding at least 14 rather than exactly 14.

Question 8 (ii)

(ii) Laura wins at least 14 games. **[2]**

Candidates who did well in this question made efficient use of the binomial distribution function in their calculator.

Candidates who did less well mistook "at least" for "greater than" in this part or made arithmetic slips when using traditional methods to calculate the probabilities.

Question 9 (i)

9 At the end of each school term at North End College all the science classes in year 10 are given a test. The marks out of 100 achieved by members of set 1 are shown in Fig. 9.

3	5
4	0 9
5	2 3 6
6	0 1 3 5 6
7	0 1 2 5 6 8 9 9
8	3 4 6 6 8 8 9
9	5 5 5 6 7

Key 5 | 2 represents a mark of 52

Fig. 9

(i) Describe the shape of the distribution. **[1]**

Some confusion between positive and negative skew was seen.

Question 9 (ii)

(ii) The teacher for set 1 claimed that a typical student in his class achieved a mark of 95. How did he justify this statement? **[1]**

A simple statement was expected, and not a mini essay. Almost every candidate gained this mark.

Question 9 (iii)

(iii) Another teacher said that the average mark in set 1 is 76. How did she justify this statement? **[1]**

A simple statement was expected, and not a mini essay. The majority of candidates gained the mark

Benson's mark in the test is 35. If the mark achieved by any student is an outlier in the lower tail of the distribution, the student is moved down to set 2.

(iv) Determine whether Benson is moved down to set 2. **[2]**

Question 9 (iv)

Candidates who did well in this question used the lower quartile and the interquartile range to determine whether Benson's mark is an outlier.

Candidates who did less well used the median in conjunction with the lower quartile.

Question 10 (i)

10 The screenshot in Fig. 10 shows the probability distribution for the continuous random variable X, where $X \sim \mathrm{N}(\mu, \sigma^2)$.

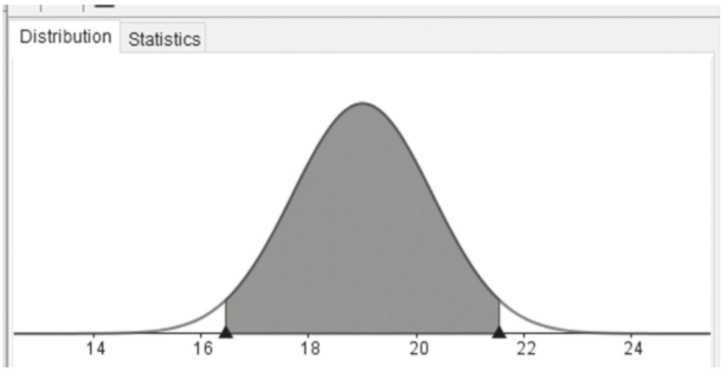


Fig. 10

The area of each of the unshaded regions under the curve is 0.025. The lower boundary of the shaded region is at 16.452 and the upper boundary of the shaded region is at 21.548.

(i) Calculate the value of μ. **[1]**

Candidates who did well in this question wrote down the mean by symmetry.

Question 10 (ii)

(ii) Calculate the value of σ^2. **[3]**

Candidates who did well on this part made efficient use of the standard Normal variable to find the variance. Candidates who did less well confused the mean with the variance, or used a wrong value for *z* (usually 1.645) to find the variance.

Question 10 (iii) (A)

(iii) Y is the random variable given by $Y = 4X + 5$.

(A) Write down the distribution of Y. **[3]**

A common error in this part was to calculate 4× 1.69 or 16× 1.69 + 5 instead of 16×1.69 .

Question 10 (iii) (B)

(B) Find P(Y > 90). **[1]**

This part required efficient use of the calculator.

Question 11 (i)

11 The discrete random variable X takes the values 0, 1, 2, 3, 4 and 5 with probabilities given by the formula

$$P(X = x) = k(x + 1)(6 - x) .$$

(i) Find the value of k. **[2]**

Candidates who did well in this question found *k* successfully and used the result in fractional form in part (ii) to calculate the requested probability.

Question 11 (ii)

In one half-term Ben attends school on 40 days. The probability distribution above is used to model X, the number of lessons per day in which Ben receives a gold star for excellent work.

(ii) Find the probability that Ben receives no gold stars on each of the first 3 days of the half-term and two gold stars on each of the next 2 days. **[2]**

The most common error was for candidates to add the two probabilities instead of multiplying.

Question 11 (iii)

(iii) Find the expected number of days in the half-term on which Ben receives no gold stars. **[2]**

There was a significant number of candidates that spoiled their answer to this part by rounding to the nearest whole number.

Question 12

12 You must show detailed reasoning in this question.

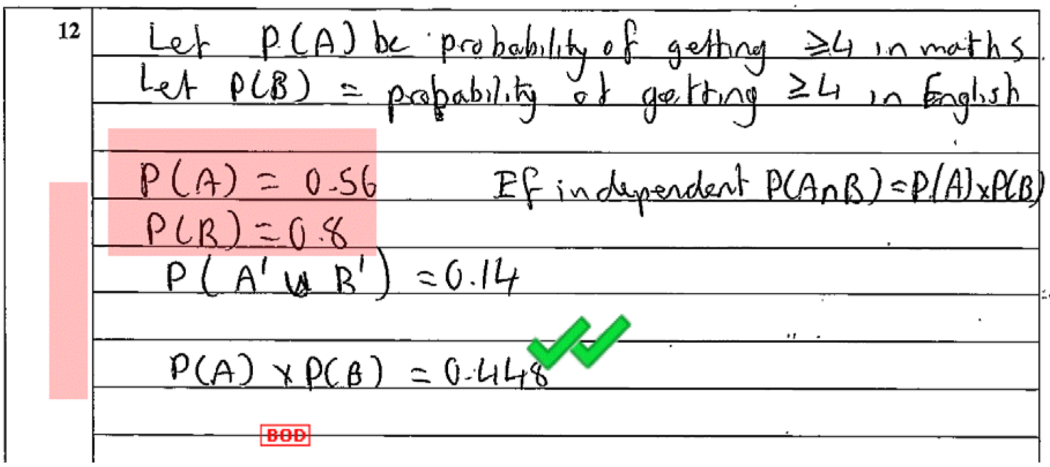
In the summer of 2017 in England a large number of candidates sat GCSE examinations in **both** mathematics **and** English. 56% of these candidates achieved at least level 4 in mathematics and 80% of these candidates achieved at least level 4 in English. 14% of these candidates did not achieve at least level 4 in either mathematics or English.

Determine whether achieving level 4 or above in English and achieving level 4 or above in mathematics were independent events. **[5]**

Candidates who did well in this question defined appropriate events and set out a clear, reasoned argument. They usually worked with a Venn diagram and clearly showed that p(*A* and *B*) ≠ p(*A*)×p(*B*) in this case.

Candidates who did less well wrote down relevant ideas but were unable to draw them together successfully. They sometimes stated results instead of giving details of the calculation.

Exemplar 1


12 Let P(A) be probability of getting ≥4 in maths
Let P(B) = probability of getting ≥4 in English

P(A) = 0.56 If independent P(A∩B) = P(A)×P(B)
P(B) = 0.8
P(A' ∪ B') = 0.14

P(A) × P(B) = 0.448

BOD

This candidate has been given BOD M1A1 for the calculation of p(*A*)×p(*B*) being correctly done, as highlighted by the statements on the LHS. The events are clearly defined.

No attempt was made to find the actual value of p(*A* and *B*), so no further progress was made.

Question 13 (i)

13 Each weekday Keira drives to work with her son Kaito. She always sets off at 8.00 a.m. She models her journey time, x minutes, by the distribution $X \sim \mathrm{N}(15, 4)$.

Over a long period of time she notes that her journey takes less than 14 minutes on 7% of the journeys, and takes more than 18 minutes on 31% of the journeys.

(i) Investigate whether Keira's model is a good fit for the data. **[3]**

Candidates who did well in this question made appropriate calculations, showed full working and explained the significance of their results.

Candidates who did less well gave incomplete reasoning and/or made slips in their calculations.

Question 13 (ii)

Kaito believes that Keira's value for the variance is correct, but realises that the mean is not correct.

(ii) Find, correct to two significant figures, the value of the mean that Keira should use in a refined model which does fit the data. **[2]**

Candidates who did less well in this question ignored the request for an answer correct to two significant figures.

Question 13 (iii)

Keira buys a new car. After driving to work in it each day for several weeks, she randomly selects the journey times for n of these days. Her mean journey time for these n days is 16 minutes. Using the refined model she conducts a hypothesis test to see if her mean journey time has changed, and finds that the result is significant at the 5% level.

(iii) Determine the smallest possible value of n. **[5]**

Candidates who did well in this question recognised that the hypothesis test was two-tailed and worked with $\sqrt{n}$ in their calculations.

Candidates who did less well worked with an incorrect z value or an incorrect form of the standard deviation.

Exemplar 2

13(i)

$P(X<14) = 0.07$ $P(X>18) = 0.31$

$\frac{14-\mu}{\sigma} \approx -1.47579$ $\frac{18-\mu}{\sigma} \approx 0.49585$

$\frac{14-15}{2} = -0.5 \neq$ ~~-1.47...~~

$\frac{18-15}{2} = 1.5$ ✓

So her model is not a good fit as her z values are far from the actual probabilities. $1.5 \neq 0.49585$

13(ii)

$\frac{14-\mu}{2} = -1.47579$

$-\mu = -2.95158 - 14$

$\mu = 16.95 = 17$ ✓

$\mu = 17$

13(iii)

$\bar{X} \sim N\left(17, \frac{2^2}{n}\right)$

$H_0: \mu = 17$ $H_1: \mu \neq 17$

where μ is mean journey time.

As $16 < 17$ and you want smallest value, find lower tail

M1 B0

$\frac{16-17}{\frac{2}{\sqrt{n}}} \leq -1.6449$ z value of $0.05 \approx -1.6449$

$\frac{16-17}{-1.6449} \leq \frac{2}{\sqrt{n}}$ M1 $n \leq \left(\frac{2}{0.607956}\right)^2$

$n \leq 10.82$

~~n = 10~~ as 11 is too large

$n = 10$

$n = 10$

Note that in part (i) the candidate has chosen a less frequently seen approach, identified in the RH column of the mark scheme.

Part (ii) is well done.

In part (iii) the wrong choice of *z* proved costly.

Question 14 (i) (A)

14 The pre-release material includes data on unemployment rates in different countries. A sample from this material has been taken. All the countries in the sample are in Europe. The data have been grouped and are shown in Fig 14.1.

Unemployment rate	0–	5–	10–	15–	20–	35–50
Frequency	15	21	5	5	2	2

Fig. 14.1

A cumulative frequency curve has been generated for the sample data using a spreadsheet. This is shown in Fig. 14.2.

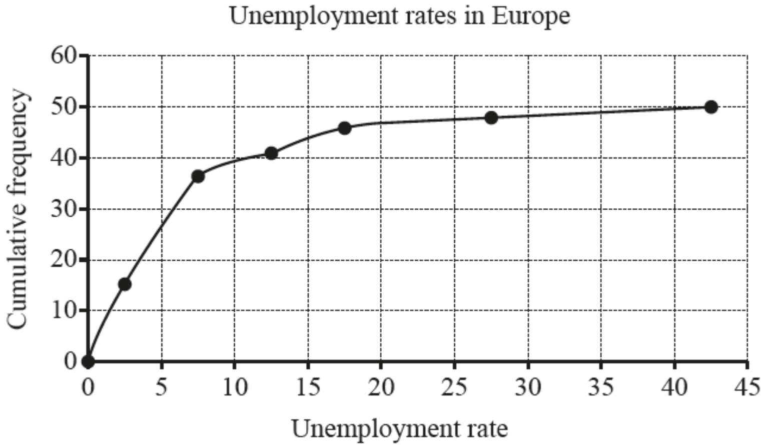


Fig. 14.2

Hodge used Fig. 14.2 to estimate the median unemployment rate in Europe. He obtained the answer 5.0. The correct value for this sample is 6.9.

(i) (*A*) There is a systematic error in the diagram.

- Identify this error.
- State how this error affects Hodge's estimate. **[2]**

Candidates who did well in this question recognised that the cumulative frequencies had been plotted at the mid-point of the intervals instead of at the upper limit.

Question 14 (i) (B)

(*B*) There is another factor which has affected Hodge's estimate.

- Identify this factor.
- State how this factor affects Hodge's estimate. **[2]**

Candidates understood that grouping the data affects the accuracy of the result and commented accordingly.

Candidates who did less well made comments about whether the points had been joined by straight lines or a curve.

Question 14 (ii)

(ii) Use your knowledge of the pre-release material to give another reason why any estimation of the median unemployment rate in Europe may be unreliable. **[1]**

Candidates who did less well based comments on general geographical or economic ideas, rather than specifically related to issues related to the estimation of median values.

Question 14 (iii)

(iii) Use your knowledge of the pre-release material to explain why it is very unlikely that the sample has been randomly selected from the pre-release material. **[1]**

Candidates who did well on part (ii) and part (iii) were familiar with the pre-release material made appropriate comments.

Question 14 (iv)

The scatter diagram shown in Fig. 14.3 shows the unemployment rate and life expectancy at birth for the 47 countries in the sample for which this information is available.

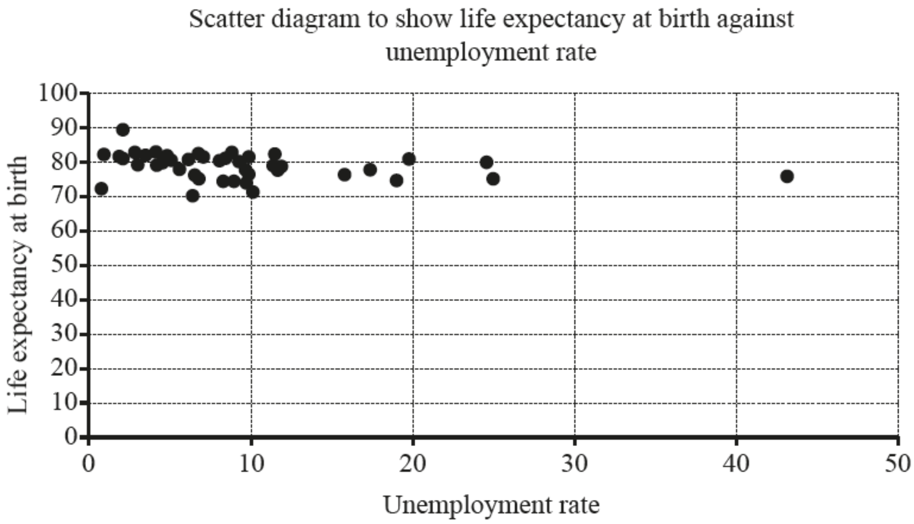


Fig. 14.3

The product moment correlation coefficient for the 47 items in the sample is – 0.2607.
The p-value associated with $r = -0.2607$ and $n = 47$ is 0.0383.

(iv) Does this information suggest that there is an association between unemployment rate and life expectancy at birth in countries in Europe? **[2]**

Candidates who did well commented on the nature of the correlation. They compared the *p*-value with a significance level and then made an appropriate deduction. Candidates who did less well compared the correlation coefficient with the *p*-value or did not comment on the association at all.

Question 14 (v)

Hodge uses the spreadsheet tools to obtain the equation of a line of best fit for this data.

(v) The unemployment rate in Kosovo is 35.3, but there is no data available on life expectancy. Is it reasonable to use Hodge's line of best fit to estimate life expectancy at birth in Kosovo? **[1]**

Candidates who did well commented on the nature of the scatter or the position of 35.3 relative to the given values to justify their comment.

Question 15

15 You must show detailed reasoning in this question.

The equation of a curve is

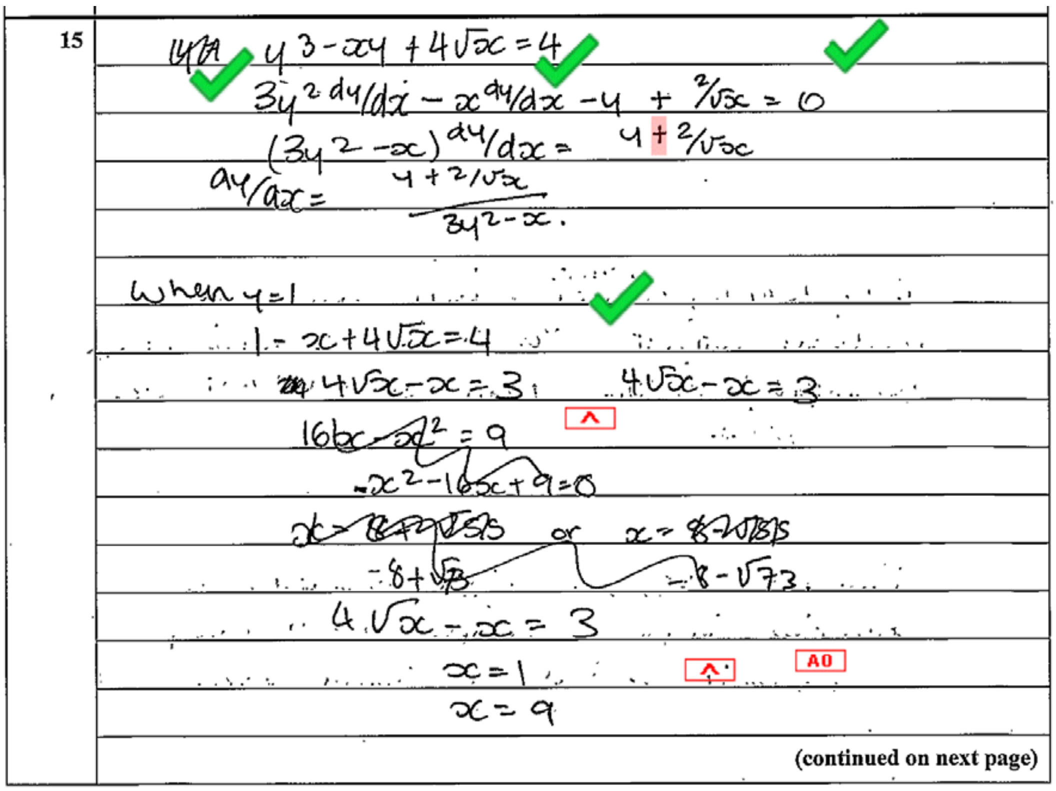
$$y^3 - xy + 4\sqrt{x} = 4 .$$

Find the gradient of the curve at each of the points where $y = 1$. **[9]**

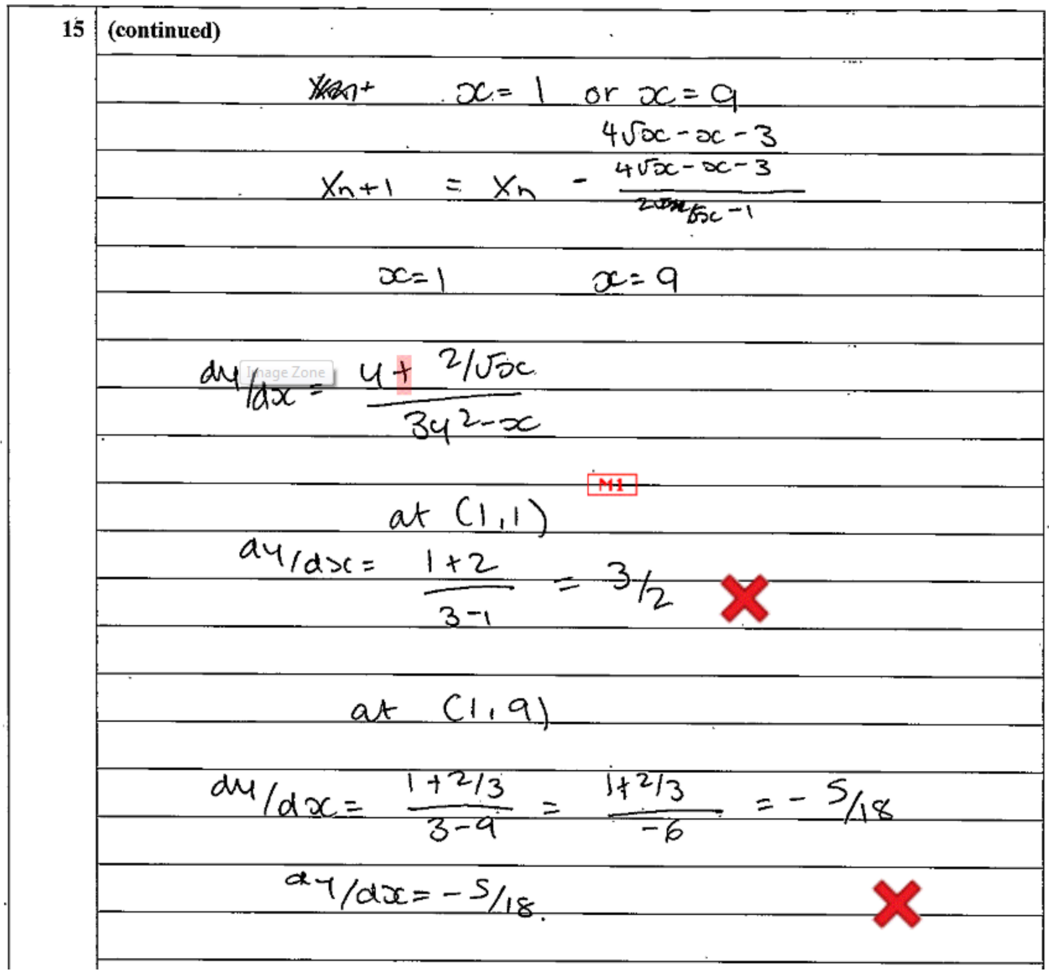
Candidates who did well in this question explained their reasoning clearly, in particular they showed how the values of *x* were obtained and how $\frac{dy}{dx}$ was obtained.

Candidates who did less well made algebraic and arithmetic slips. They did not fully explain their reasoning, especially when finding the values of *x*.

Exemplar 3


15
$y^3 - xy + 4\sqrt{x} = 4$
$3y^2\, dy/dx - x\, dy/dx - y + 2/\sqrt{x} = 0$
$(3y^2 - x)\, dy/dx = y + 2/\sqrt{x}$
$dy/dx = \dfrac{y + 2/\sqrt{x}}{3y^2 - x}$
When $y = 1$
$1 - x + 4\sqrt{x} = 4$
$4\sqrt{x} - x = 3$ $4\sqrt{x} - x = 3$
$16x - x^2 = 9$
$-x^2 - 16x + 9 = 0$
$x =$ ... or $x =$ ...
$-8 + \sqrt{73}$ $-8 - \sqrt{73}$
$4\sqrt{x} - x = 3$
$x = 1$ AO
$x = 9$

(continued on next page)



15 (continued)

~~Xn+1~~ $x = 1$ or $x = 9$

$$x_{n+1} = x_n - \frac{4\sqrt{x} - x - 3}{2\sqrt{x} - 1}$$

$x = 1$ $x = 9$

$$dy/dx = \frac{4 + 2/\sqrt{x}}{3y^2 - x}$$

M1

at $(1, 1)$

$$dy/dx = \frac{1+2}{3-1} = 3/2$$ ✗

at $(1, 9)$

$$dy/dx = \frac{1 + 2/3}{3 - 9} = \frac{1 + 2/3}{-6} = -5/18$$

$dy/dx = -5/18.$ ✗

This candidate differentiated correctly, but made a sign error when rearranging, which cost accuracy marks at the end. Note that detailed reasoning was required in this question. It was not clear how the (correct) values of *x* were obtained so A marks were withheld.

Question 16 (i)

16 In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for *t* hours. Over the course of the year she realises that her percentage mark for a test, *p*, may be modelled by the following formula, where *A*, *B* and *C* are constants.

$$p = A - B(t - C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.
- Aaishah had a mark of 22 when she didn't spend any time revising.

(i) Find the values of *A*, *B* and *C*. **[3]**

Candidates who did well in this question recognised that the maximum value of *p* has to be 62, and that this occurs when $t = 2$, thus obtaining *A* and *C*. The value of *B* soon follows.

This who did less well wrote down three equations in three variables and often went astray.

Question 16 (ii)

(ii) According to the model, if Aishah revises for 45 minutes on the night before the test, what mark will she achieve? **[2]**

Candidates who did well made the correct substitution in their formula.

Candidates who did less well substituted $t = 45$.

Question 16 (iii)

(iii) What is the maximum amount of time that Aaishah could have spent revising for the model to work? **[2]**

Candidates who did well understood that the value of p couldn't be negative and obtained a value for t accordingly.

Candidates who did less well worked with < instead of = or >.

Question 16 (iv)

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p.

$$p = 22 + 68(1 - e^{-0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
p	59	84	89

Fig. 16

(iv) Verify that the data is consistent with the new formula. **[2]**

Candidates who did well made correct substitutions and provided a supporting comment.

Candidates who did less well neglected to substitute all three values or made no comment on what their calculations showed.

Question 16 (v)

(v) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future. Determine whether or not this is good advice. **[2]**

Candidates who did well evaluated p at $t = 12$ and either at $t = 5$ or a value in between 5 and 12 and commented appropriately.

Candidates who did less well made no supporting calculations but supplied a comment, or vice versa.

Question 17 (i)

17 **(i)** Express $\frac{(x^2-8x+9)}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

Candidates who did well recognised the correct form of partial fractions and were able to work successfully to find the coefficients.

Candidates who did less well made algebraic slips in clearing the fractions or made slips in arithmetic when finding the coefficients.

Question 17 (ii)

(ii) Express y in terms of x given that

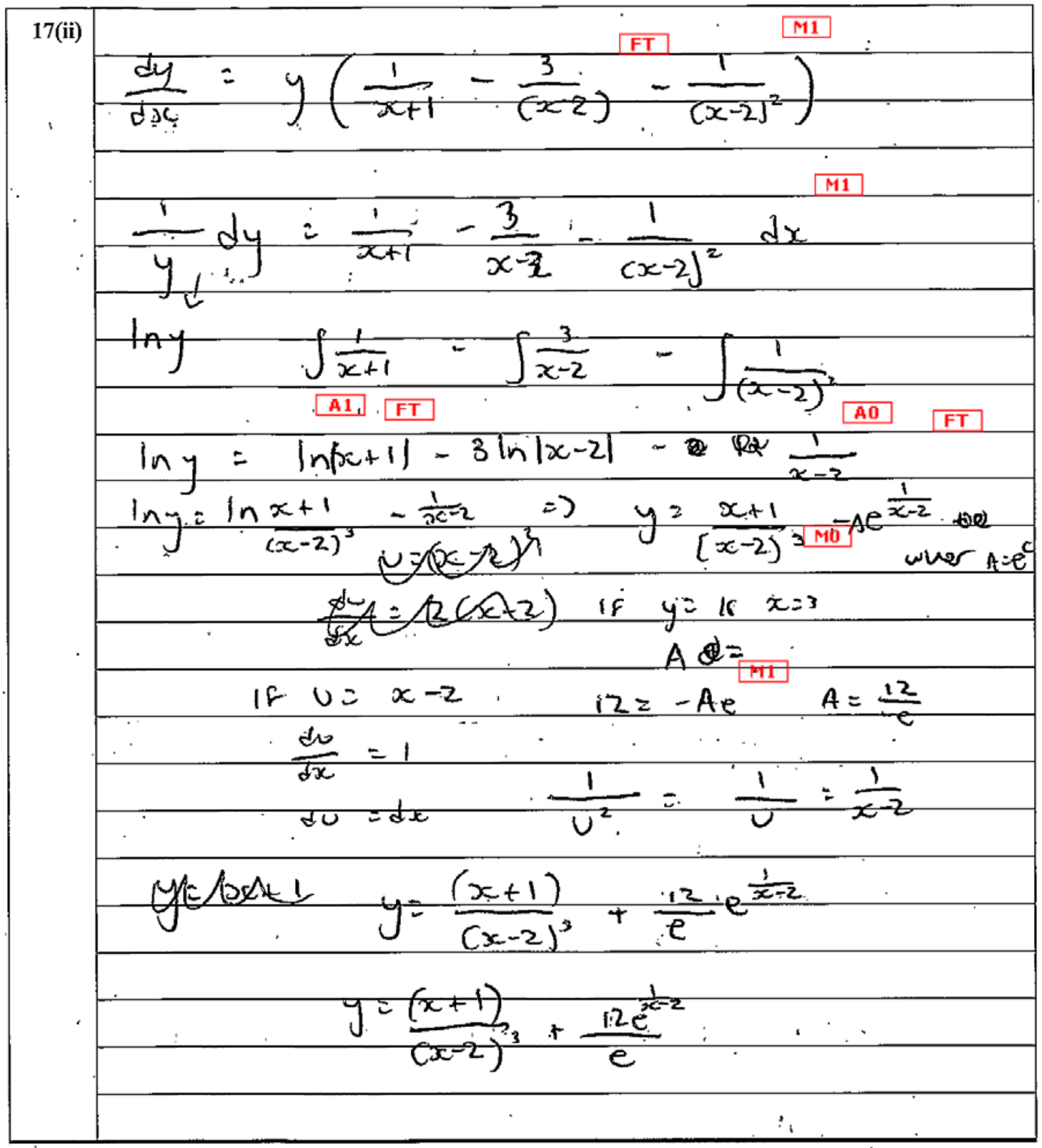
$$\frac{dy}{dx}=\frac{y(x^2-8x+9)}{(x+1)(x-2)^2} \text{ and } y=16 \text{ when } x=3.$$

[7]

Candidates who did well recognised the need to use their result from part (i). They separated the variables successfully and were then able to integrate and substitute the values of *x* and *y* to find the constant of integration. Candidates who did very well were able to go on and find a correct expression for *y*.

Candidates who did less well rearranged incorrectly when they attempted to separate the variables, or were unable to integrate the quadratic term correctly. They made slips in exponentiating both sides of their equation, usually assuming that the operation is distributive.

Exemplar 4



In this response FT marks have been credited for the use of their partial fractions and separation of variables. One A mark has been credited FT, but the integration of the quadratic term went astray.

The exponentiation of both sides was incorrect, but in spite of this, the method mark for substitution was subsequently earned.

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Exemplar Candidate Work

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/02 Summer 2018 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification http://www.ocr.org.uk/Images/308740-specification-accredited-a-level-gce-mathematics-b-mei-h640.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1

1 Show that $\sqrt{27} + \sqrt{192} = a\sqrt{b}$, where a and b are prime numbers to be determined. **[2]**

Exemplar 1 — 2 marks

1

$\sqrt{27} = \sqrt{9 \times 3}$ $\qquad$ $\sqrt{192} = \sqrt{64 \times 3}$

$= 3\sqrt{3}$ $\qquad$ $= 8\sqrt{3}$

so $\sqrt{27} + \sqrt{192} = 3\sqrt{3} + 8\sqrt{3}$

$= 11\sqrt{3}$ ✓

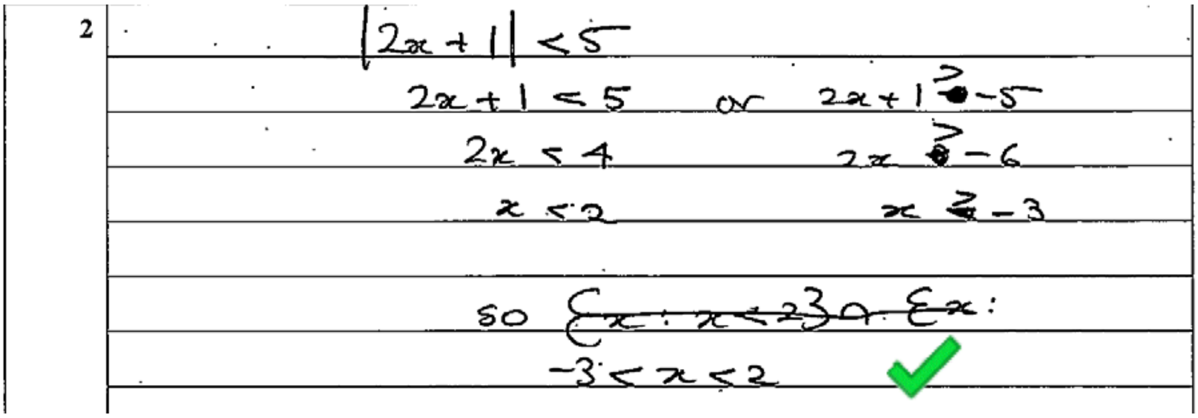
Examiner commentary

This response is clearly set out and fully correct, thus earning both marks. The overwhelming majority of responses to this question were of a similar standard and were credited both marks.

Question 2

2 Solve the inequality $|2x+1| < 5$. **[3]**

Exemplar 1 3 marks


2

$|2x+1| < 5$

$2x+1 < 5$ or $2x+1 > -5$

$2x < 4$ $\quad 2x > -6$

$x < 2$ $\quad x > -3$

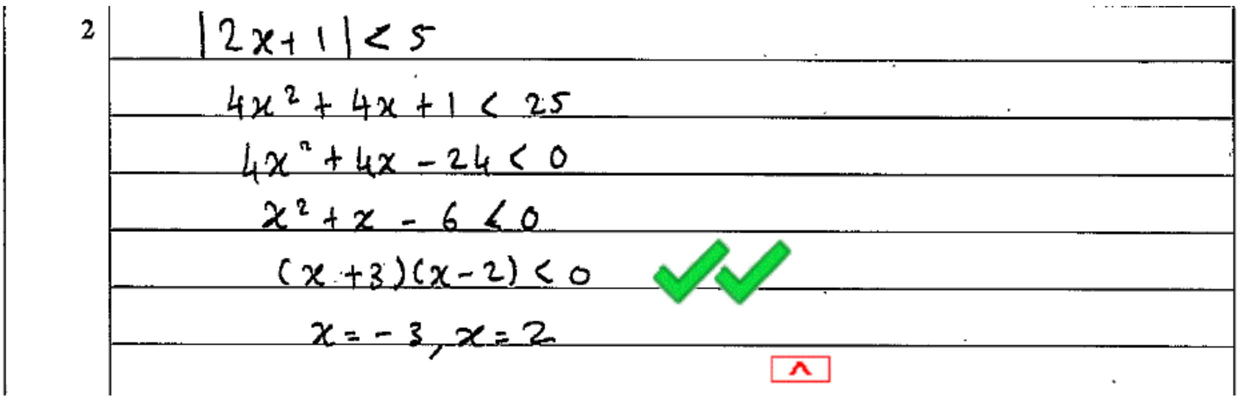
so ~~$\{x : x < 2\}$ or $\{x :$~~

$-3 < x < 2$

Examiner commentary

This fully correct response was well laid out and was credited all three marks.

Exemplar 2 2 marks


2

$|2x+1| < 5$

$4x^2 + 4x + 1 < 25$

$4x^2 + 4x - 24 < 0$

$x^2 + x - 6 < 0$

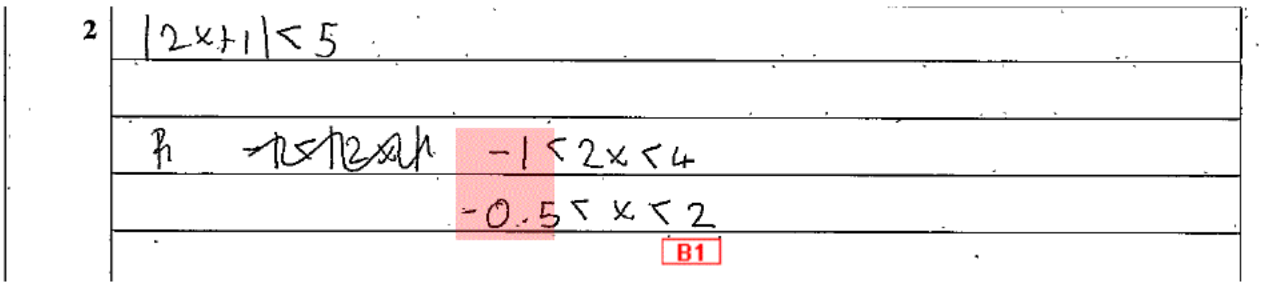
$(x+3)(x-2) < 0$

$x = -3, x = 2$

Examiner commentary

The critical values were identified by squaring both sides and solving the quadratic. However, the inequality was not stated so the final mark was withheld.

Exemplar 3 1 mark



Examiner commentary

The method mark was not credited because the work with the lower tail is incorrect. However, B1 was earned for correctly identifying $x < 2$.

Question 3(i) and (ii)

3 The probability that Chipping FC win a league football match is $P(W) = 0.4$.

(i) Calculate the probability that Chipping FC fail to win each of their next two league football matches. **[1]**

The probability that Chipping FC lose a league football match is $P(L) = 0.3$.

(ii) Explain why $P(W) + P(L) \neq 1$. **[1]**

Exemplar 1

(i) 1 mark

3(i)	P(Don't win) = 0.6
	0.6×0.6 = 0.36

(ii) 1 mark

3(ii)	Because they can also draw a game. So P(W) + P(L) + P(D) = 1

Examiner commentary

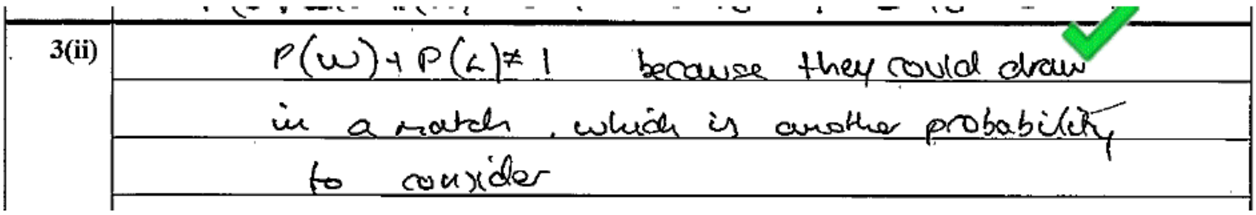
This response earned full marks in both parts of the question. Most candidates did very well in answering this question.

Exemplar 2

(i) 0 marks

3(i)	~~p(lose) = 1 − P(W) = 1 − 0.4~~
	~~= 0.6~~ ~~x is game lost~~
	~~P(x=2) = 0.6 × 0.6 = 0.36~~ ~~loss AND loss = multiply~~
	P(W) and P(W) = 0.4^2 = 0.16 1 − 0.16 = 0.84 ✗

(ii) 1 mark



3(ii) $P(W) + P(L) \neq 1$ because they could draw in a match, which is another probability to consider

Examiner commentary

In part (i), the probability was squared and then subtracted from 1. The order of calculation should have been reversed. No mark was credited here.

The explanation in part (ii) is acknowledged an alternate outcome and was credited B1.

Question 4(i) and (ii)

4 A survey of the number of cars per household in a certain village generated the data in Fig. 4.

Number of cars	0	1	2	3	4
Number of households	8	22	31	27	7

Fig. 4

(i) Calculate the mean number of cars per household. **[1]**

(ii) Calculate the standard deviation of the number of cars per household. **[1]**

Exemplar 1

(i) 1 mark

4(i)

$$\bar{x} = \frac{(8\times 0) + (1\times 22) + (2\times 31) + (3\times 27) + 4(7)}{8 + 22 + 31 + 27 + 7}$$

$= 2.63$ cars

(ii) 1 mark

4(ii)

$$s = \sqrt{\frac{\Sigma x^2 - n\bar{x}^2}{n-1}} = \sqrt{\frac{501 - 95(2.03)^2}{94}}$$

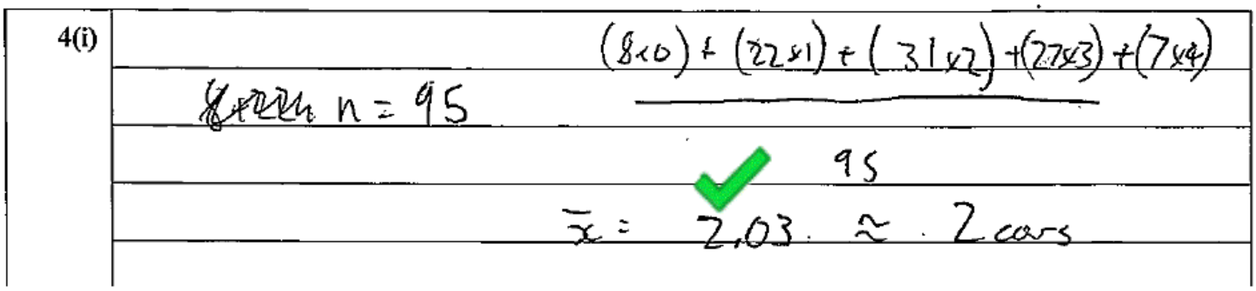
$= 1.08$ (3sf)

Examiner commentary

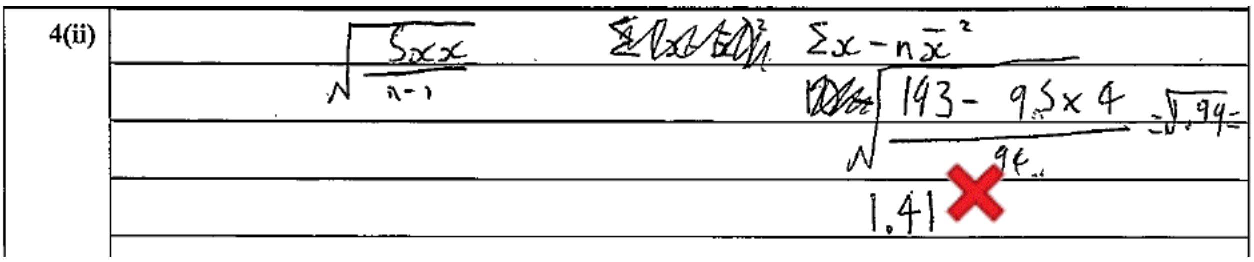
Full marks were credited for a fully correct response. It is worth noting that candidates were simply expected to use their calculators efficiently, and a correct statement of the answers was all that was required.

Exemplar 2

(i) 1 mark



(ii) 0 marks



Examiner commentary

It appears that the candidate opted to calculate both values laboriously instead of using the appropriate calculator function. A slip in the second calculation has cost what was considered to be a straightforward mark.

Question 5(i)(*A*), (i)(*B*) and (ii)

5 **(i)** (*A*) Sketch the graph of $y = 3^x$. **[1]**

(*B*) Give the coordinates of any intercepts. **[1]**

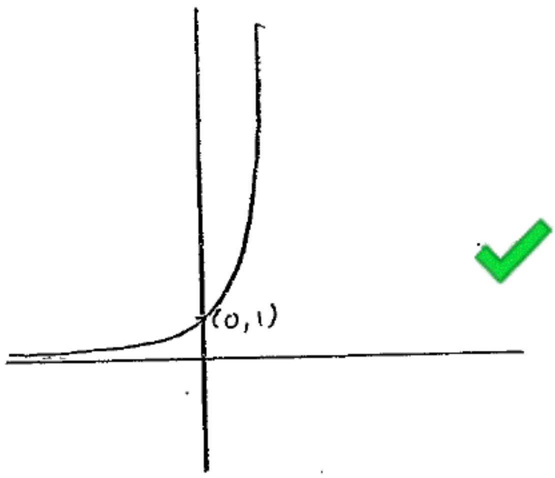
The curve $y = \mathrm{f}(x)$ is the reflection of the curve $y = 3^x$ in the line $y = x$.

(ii) Find $\mathrm{f}(x)$. **[1]**

Exemplar 1

(i)(*A*) 1 mark

5(i)(*A*)



(i)(*B*) 1 mark

5(i)(*B*)	(0, 1)

(ii) 1 mark

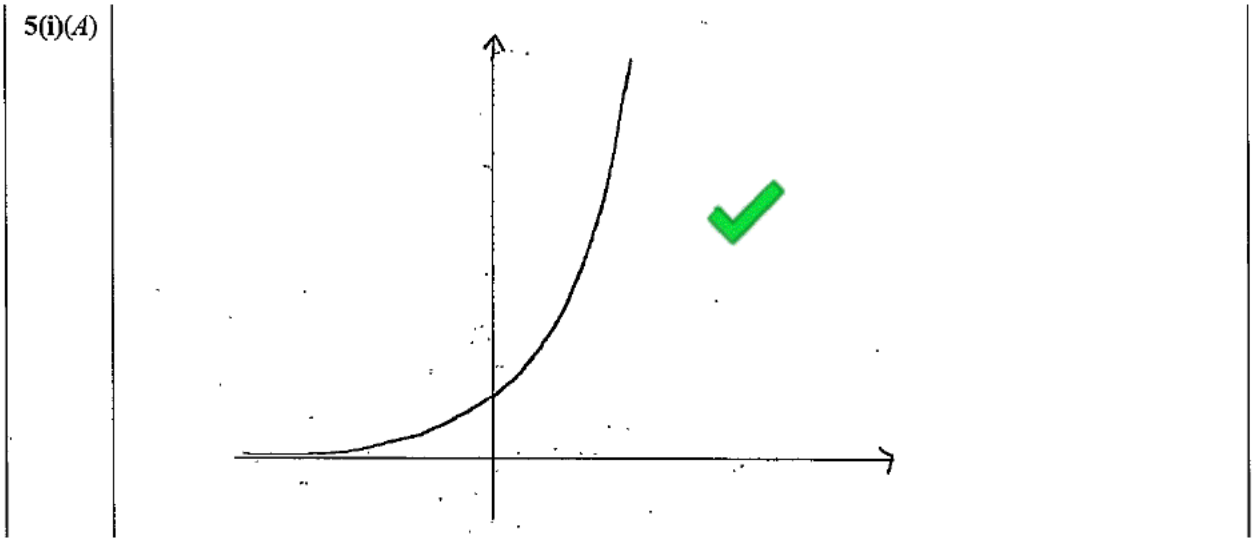
5(ii)	$x = 3^y$
	$\log_3 x = y$

Examiner commentary

This fully correct and well laid out response was credited all three marks.

Exemplar 2

(i)(*A*) 1 mark



(i)(*B*) 1 mark

5(i)(*B*) $(0, 1)$

(ii) 0 marks

5(ii)

$y = 3^x$

$x = 3^y$

$\ln x = y \ln 3$

$\ln|x - 3| = y$

Examiner commentary

Parts (i) and (ii) were fully correct. It is worth noting that $f(x) = \dfrac{\ln x}{\ln 3}$ would have been credited the mark.

Question 6(i) and (ii)

6 **(i)** Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$, where $0 < \alpha < \frac{\pi}{2}$. **[3]**

(ii) Write down the range of the function

$$f(x) = 12 + 7\cos x - 24\sin x, \quad 0 \leqslant x \leqslant 2\pi.$$

[2]

Exemplar 1

(i) 3 marks

6(i)

$7\cos x - 24\sin x \equiv R\cos(x + \alpha)$

$\equiv R\cos x\cos\alpha - R\sin x\sin\alpha$

$7\cos x = R\cos x\cos\alpha \qquad -24\sin x = -R\sin x\sin\alpha$

$R\cos\alpha = 7 \qquad R\sin\alpha = 24$

$R = \sqrt{7^2 + 24^2} = 25$

$\alpha = \tan^{-1}\left(\frac{24}{7}\right) = 1.287 \text{ (4sf)}$

$7\cos x - 24\sin x = 25\cos(x + 1.287)$

(ii) 2 marks

6(ii)

$f(x) = 12 + 7\cos x - 24\sin x$

$= 25\cos(x + 1.287) + 12$

$\cos\theta$: Range $= -1 \leqslant f(x) \leqslant 1$

$25\cos\theta = -25 \leqslant f(x) \leqslant 25$

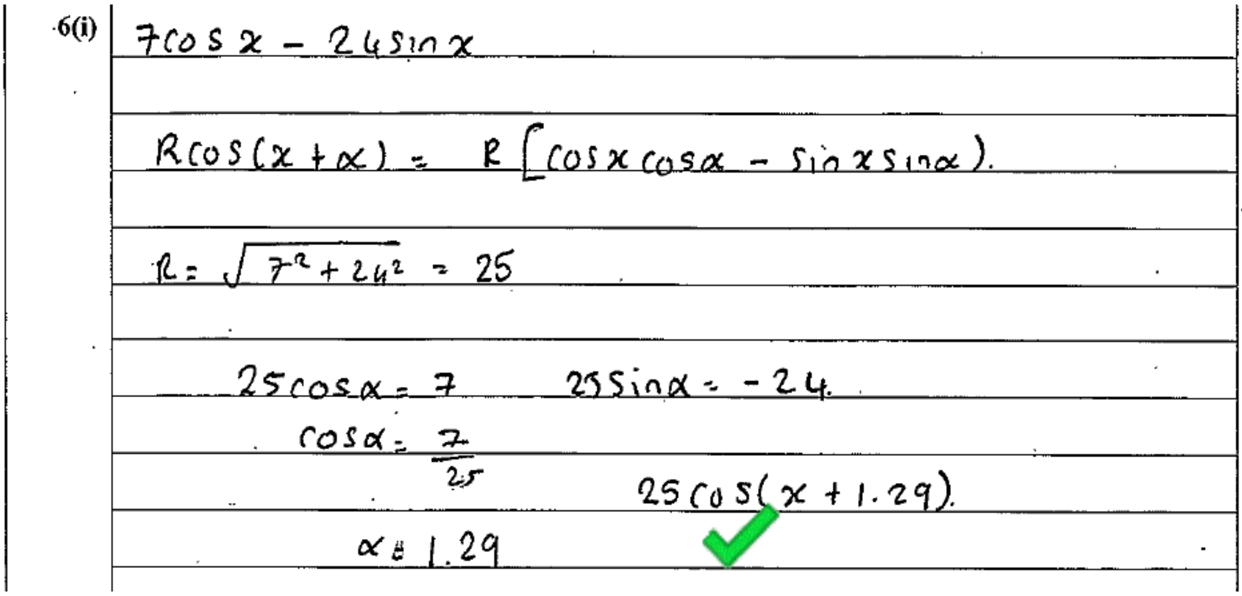
$25\cos\theta + 12$: Range $= -13 \leqslant f(x) \leqslant 37$

Examiner commentary

This is a fully correct solution. In part (ii), the candidate has used a logical progression to identify the range to minimise the risk of mistake.

Exemplar 2

(i) 3 marks


6(i)

$7\cos x - 24\sin x$

$R\cos(x+\alpha) = R[\cos x\cos\alpha - \sin x\sin\alpha)$.

$R = \sqrt{7^2+24^2} = 25$

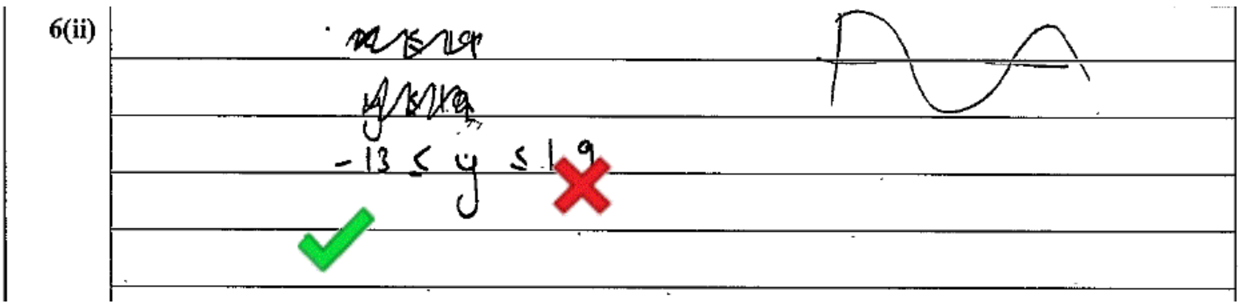
$25\cos\alpha = 7 \qquad 25\sin\alpha = -24$.

$\cos\alpha = \frac{7}{25}$

$25\cos(x+1.29)$.

$\alpha = 1.29$

(ii) 1 mark


6(ii)

$-13 \le y \le 19$

Examiner commentary

Part (i) was fully correct. In part (ii), M1 was allowed for correctly identifying the lower value.

Exemplar 3

(i) 3 marks

6(i)

$7\cos x - 24\sin x \qquad R\cos(x+\alpha)$

$y = R[\cos\theta\cos\alpha - \sin\theta\sin\alpha]$

$7 = R\cos\alpha \qquad 24 = R\sin\alpha$

$\frac{24}{7} = \frac{R\sin\alpha}{R\cos\alpha} = \tan\alpha \qquad 1.29$ (3SF) (Radians)

$\tan^{-1}\left(\frac{24}{7}\right) = 1.287002218$

$(R\cos\alpha)^2 + (R\sin\alpha)^2 = 24^2 + 7^2$

$R^2(\cos^2\alpha + \sin^2\alpha) = 625$

$R^2 = 625 \qquad R = 25$

$\therefore 7\cos x - 24\sin x = 25\cos(x + 1.29)$

(ii) 0 marks

6(ii)

@ $x = 0 \quad f(x) = 19$

@ $x = 2\pi \quad f(x) = 19$

@ $x = \frac{3\pi}{2}$

Range $= 0 \leq x \leq 19$

Examiner commentary

Part (i) was well answered. However, in part (ii) the candidate considered the end points of the domain instead of the possible values of the cosine function.

Question 7

7 Find $\int\left(4\sqrt{x}-\frac{6}{x^3}\right)\mathrm{d}x$. **[4]**

Exemplar 1 3 marks

7

-2.6255

$$\int\left(4\sqrt{x}-\frac{6}{x^3}\right)dx$$

$$\int\left(4x^{\frac{1}{2}}-6x^{-3}\right)dx$$

$$=\frac{4x^{\frac{3}{2}}}{\frac{3}{2}}-\frac{6x^{-2}}{-2}$$

$$=\frac{8}{3}x^{\frac{3}{2}}--3x^{-2}$$

$$=\frac{8}{3}x^{\frac{3}{2}}+\frac{3}{x^2}$$

^

Examiner commentary

The integration was carried out successfully, but the final accuracy mark was withheld because no constant of integration was included.

Question 8(i) and (ii)

8 Every morning before breakfast Laura and Mike play a game of chess. The probability that Laura wins is 0.7. The outcome of any particular game is independent of the outcome of other games. Calculate the probability that, in the next 20 games,

(i) Laura wins exactly 14 games, **[2]**

(ii) Laura wins at least 14 games. **[2]**

Exemplar 1

(i) 2 marks

8(i)

$X \sim B(20, 0.7)$

X is the ~~probability~~ games Laura wins

$P(X=14) = 0.19163898$

$= 0.192$ (3sf) ✓

(ii) 1 mark

8(ii)

$P(X \geq 14) = 1 - P(X < 14)$

$= 1 - P(X \leq 13)$ ✓

$= 1 - 0.39199018$

$= 0.60800982$

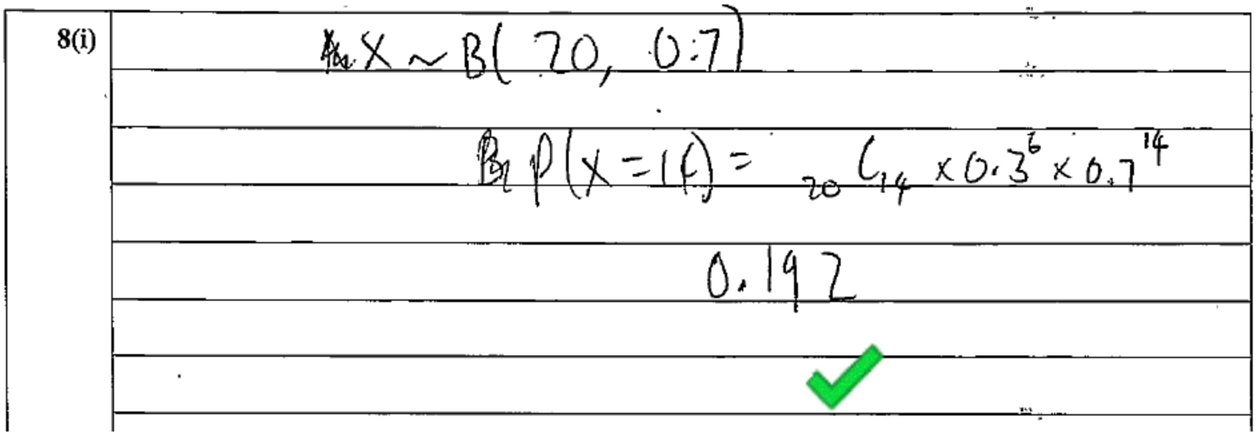
Examiner commentary

It is worth noting that efficient use of the appropriate calculator functions is expected in this question.

This candidate has answered the question well, but slipped up in copying the answer from the calculator. The level of precision quoted by the candidate is not expected in this type of question. Rounding the answer to 3 significant figures might have made a slip such as this less likely.

Exemplar 2

(i) 2 marks



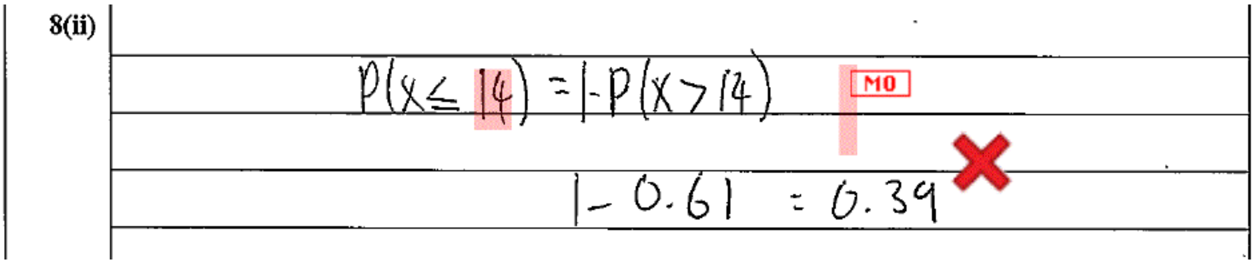
8(i)

$X \sim B(20, 0.7)$

$P(X = 14) = {}_{20}C_{14} \times 0.3^6 \times 0.7^{14}$

0.192

(ii) 0 marks



8(ii)

$P(X \le 14) = 1 - P(X > 14)$ M0

$1 - 0.61 = 0.39$

Examiner commentary

Part (i) was well done and earned both marks. The response to part (ii) earned no marks as both the method and the value quoted were incorrect.

Question 9(i), (ii), (iii) and (iv)

9 At the end of each school term at North End College all the science classes in year 10 are given a test. The marks out of 100 achieved by members of set 1 are shown in Fig. 9.

3	5
4	0 9
5	2 3 6
6	0 1 3 5 6
7	0 1 2 5 6 8 9 9
8	3 4 6 6 8 8 9
9	5 5 5 6 7

Key 5 | 2 represents a mark of 52

Fig. 9

(i) Describe the shape of the distribution. **[1]**

(ii) The teacher for set 1 claimed that a typical student in his class achieved a mark of 95. How did he justify this statement? **[1]**

(iii) Another teacher said that the average mark in set 1 is 76. How did she justify this statement? **[1]**

Benson's mark in the test is 35. If the mark achieved by any student is an outlier in the lower tail of the distribution, the student is moved down to set 2.

(iv) Determine whether Benson is moved down to set 2. **[2]**

Exemplar 1

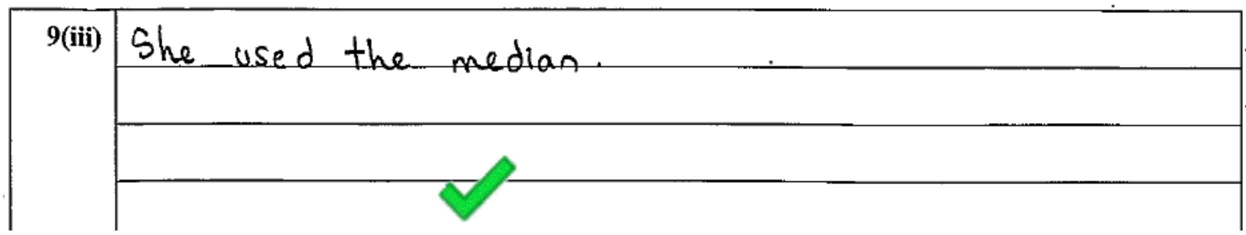
(i) 1 mark

9(i) Negative skew.

(ii) 1 mark

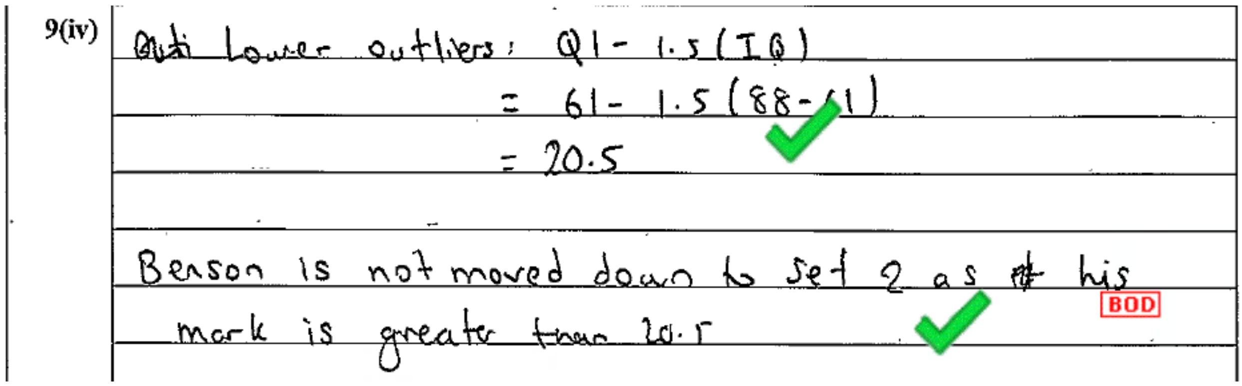
9(ii) The teacher used the mode.

(iii) 1 mark



9(iii) She used the median.

(iv) 2 marks



9(iv) ~~Outli~~ Lower outliers: $Q1 - 1.5(IQ)$

$= 61 - 1.5(88-61)$

$= 20.5$

Benson is not moved down to set 2 as his mark is greater than 20.5

BOD

Examiner commentary

This response was credited full marks. In part (iv), a direct comparison between 35 and 20.5 was expected, rather than an allusion to Benson's mark, so the final A1 is BOD in this case.

Question 10(i), (ii), (iii)(*A*) and (iii)(*B*)

10 The screenshot in Fig. 10 shows the probability distribution for the continuous random variable X, where $X \sim \mathrm{N}(\mu, \sigma^2)$.

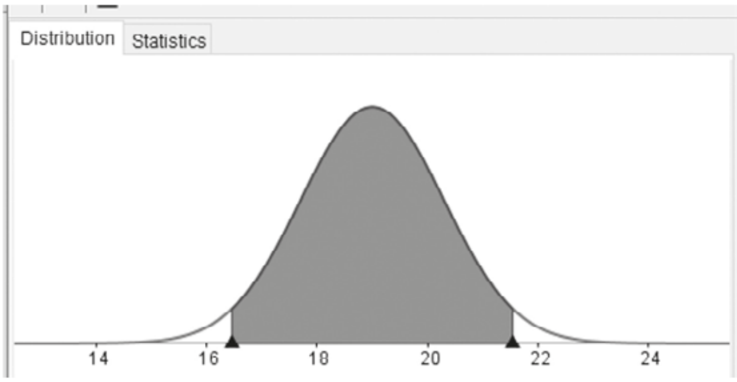


Fig. 10

The area of each of the unshaded regions under the curve is 0.025. The lower boundary of the shaded region is at 16.452 and the upper boundary of the shaded region is at 21.548.

(i) Calculate the value of μ. **[1]**

(ii) Calculate the value of σ^2. **[3]**

(iii) Y is the random variable given by $Y = 4X + 5$.

(*A*) Write down the distribution of Y. **[3]**

(*B*) Find $\mathrm{P}(Y > 90)$. **[1]**

Exemplar 1

(i) 1 mark

10(i)

$$\frac{21.548 - 16.452}{2} = 2.548$$

$16.452 + 2.548 = 19 \quad \mu = 19$

(ii) 3 marks

10(ii)

$z = \dfrac{x - \mu}{\sigma}$

$z = -1.95996$ ✓

$-1.95996 = \dfrac{16.452 - 19}{\sigma}$

$\sigma = \dfrac{16.452 - 19}{-1.95996} = 1.3$ ✓ ✓

$1.3^2 = 1.690$ ~~[crossed out]~~ 3dp.

(iii)(*A*) 3 marks

10(iii)(*A*)

$4 \times 19 + 5 = 81$

$4 \times$ ~~[crossed out]~~ 1.3 = ~~[crossed out]~~ 5.2 $\quad \sigma = 1.3$

✓

$Y \sim N(81,$ ~~[crossed out]~~ $5.2^2)$ ✓

$\downarrow$

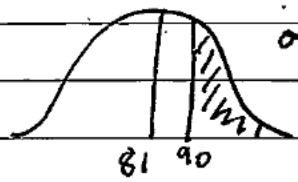
$Y \sim N(81, 27.04)$

(iii)(*B*) 1 mark

10(iii)(*B*)

$P(Y > 90) = 1 - P(Y < 90)$

$= 1 -$ ~~[crossed out]~~ 0.958



$\sigma =$ ~~[crossed out]~~ 5.2

~~0.0426 3sf.~~

$= 0.0417$ ~~[crossed out]~~ 3sf. ✓

Examiner commentary

This response was credited full marks. The work is well set out, and in spite of a number of crossings out, the work is easy to follow.

Question 11(i), (ii) and (iii)

11 The discrete random variable X takes the values 0, 1, 2, 3, 4 and 5 with probabilities given by the formula

$$P(X = x) = k(x + 1)(6 - x) .$$

(i) Find the value of k. **[2]**

In one half-term Ben attends school on 40 days. The probability distribution above is used to model X, the number of lessons per day in which Ben receives a gold star for excellent work.

(ii) Find the probability that Ben receives no gold stars on each of the first 3 days of the half-term and two gold stars on each of the next 2 days. **[2]**

(iii) Find the expected number of days in the half-term on which Ben receives no gold stars. **[2]**

Exemplar 1

(i) 2 marks

11(i)

0	1	2	3	4
$k(1)(6)$	$k(2)(5)$	$k(3)(4)$	$k(4)(3)$	$k(5)(2)$
$6k$	$10k$	$12k$	$12k$	$10(k)$

5

$k(6)(1)$

$6k$

$6k + 10k + 12k + 12k + 10k + 6k = 1$

$k(6 + 10 + 12 + 12 + 10 + 6) = 1$

$k = \frac{1}{56}$ ✓

(ii) 2 marks

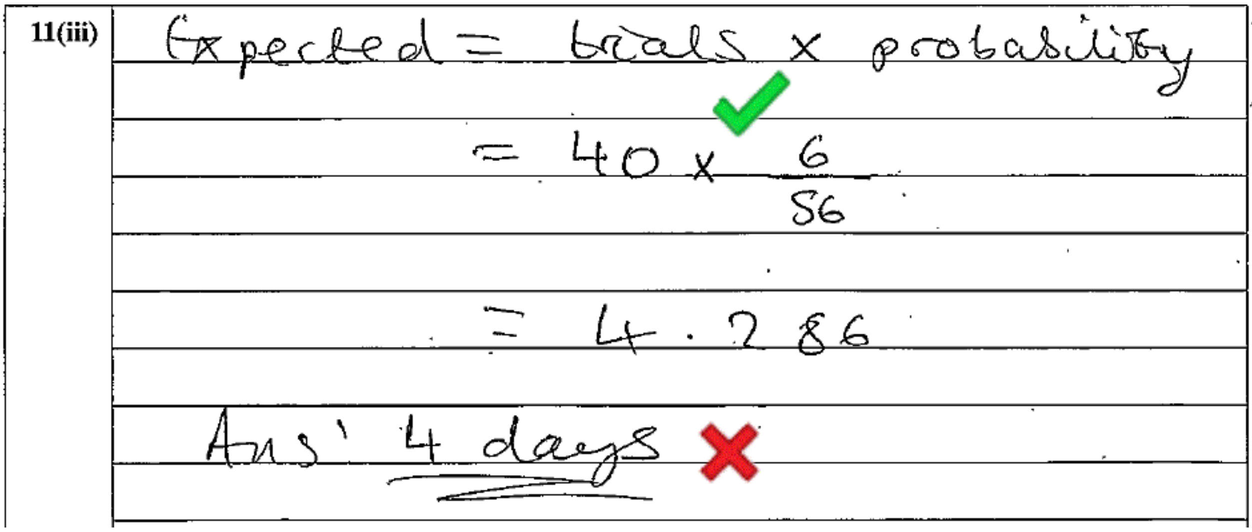
11(ii)

$P(X=0) \quad \frac{6}{56}$

$P(X=2) = \frac{12}{56}$ ✓

$\left(\frac{6}{56}\right)^3 \times \left(\frac{12}{56}\right)^2 = 5.648 \times 10^{-5}$ ✓ BOD

(iii) 1 mark



11(iii) Expected = trials × probability

$= 40 \times \frac{6}{56}$

$= 4.286$

Ans' 4 days

Examiner commentary

Parts (i) and (ii) are correctly answered and well laid out, with each response scoring 2 out of 2. In part (iii), the initial work is correct, but the final A1 is credited for the final answer. In this case, the candidate, like many others, rounded to the nearest whole number and therefore lost the final mark.

Question 12

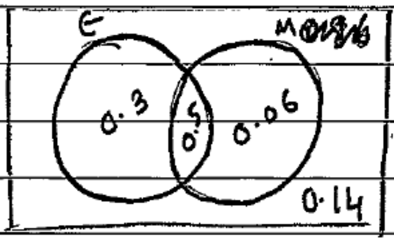
12 You must show detailed reasoning in this question.

In the summer of 2017 in England a large number of candidates sat GCSE examinations in **both** mathematics **and** English. 56% of these candidates achieved at least level 4 in mathematics and 80% of these candidates achieved at least level 4 in English. 14% of these candidates did not achieve at least level 4 in either mathematics or English.

Determine whether achieving level 4 or above in English and achieving level 4 or above in mathematics were independent events. **[5]**

Exemplar 1 5 marks

12 $P(E) = 0.8$ $P(M) = 0.56$



$0.86 = 0.8 + 0.56 - P(E \cap M)$.

$P(E \cap M) = 0.5$

$P(E) \times P(M) = 0.8 \times 0.56 = 0.448$

$0.448 \neq 0.5$ ∴ not independent events.

Examiner commentary

Candidates who did well in this question defined appropriate events and set out a clear, reasoned argument. As in this exemplar, successful responses included a Venn diagram and clearly showed that p(*A* and *B*) was not equal to p(*A*) x p(*B*).

Exemplar 2 — 0 marks

12

Maths Level 4 ≥ 56%

~~Maths Level 4 ≥ 8 English Level 4 ≥ 8~~

$E|M$ = ~~80~~ 0.8 M = ~~56~~ 0.56

$\bar{E}|\bar{M} = 0.14$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$P(E \cap M) = 0.448$ ^

$P(M) = 0.56$

$\frac{0.448}{0.56} = 0.8$ therefore they are independant

~~$P(A \cup B) = P(A) + P(B)$~~ $- P(A \cap B)$

Examiner commentary

0.448 is seen, but there is no supporting calculation and it is misidentified as p(A and B), so no credit is given.

Question 13(i), (ii) and (iii)

13 Each weekday Keira drives to work with her son Kaito. She always sets off at 8.00 a.m. She models her journey time, x minutes, by the distribution $X \sim \mathrm{N}(15, 4)$.

Over a long period of time she notes that her journey takes less than 14 minutes on 7% of the journeys, and takes more than 18 minutes on 31% of the journeys.

(i) Investigate whether Keira's model is a good fit for the data. **[3]**

Kaito believes that Keira's value for the variance is correct, but realises that the mean is not correct.

(ii) Find, correct to two significant figures, the value of the mean that Keira should use in a refined model which does fit the data. **[2]**

Keira buys a new car. After driving to work in it each day for several weeks, she randomly selects the journey times for n of these days. Her mean journey time for these n days is 16 minutes. Using the refined model she conducts a hypothesis test to see if her mean journey time has changed, and finds that the result is significant at the 5% level.

(iii) Determine the smallest possible value of n. **[5]**

Exemplar 1

(i) 3 marks

13(i)

$X \sim N(15, 4)$.

$P(X < 14) = P(Z < \frac{-1}{2}) = 0.309 \therefore 30.9\%$

$P(X > 18) = P(Z > \frac{3}{2}) = 0.06681 \quad 6.68\%$

Keira's model is not a good fit for the data

(ii) 1 mark

13(ii)

$X \sim N(\mu, 4)$ $\mu = \mu$, $\sigma = 2$

$P(X < 14) = 0.07$

$P\left(Z < \frac{14-\mu}{2}\right) = 0.07$

$\frac{14-\mu}{2} = -1.475791$

$\mu = 16.951582$

$\mu = 17.0$ (3sf). A0

(iii) 5 marks

13(iii)

$\bar{x} = 16$

$P\left(\bar{Z} < \frac{16-17}{\frac{2}{\sqrt{n}}}\right) < 0.025$

$P\left(Z < \frac{-1}{\frac{2}{\sqrt{n}}}\right) = 0.025$

$\frac{-1}{\frac{2}{\sqrt{n}}} = -1.959964$ (A)

$-1 = \frac{2A}{\sqrt{n}}$

$-\sqrt{n} = 2A$

$\therefore n = 15.3658$

$\therefore n = 16$.

Examiner commentary

The response to all three parts of this question was very good indeed. The question asked for the answer to be given to 2 significant figures in part (ii), and an easy mark was lost by giving the answer to a different precision.

Exemplar 2

(i) 3 marks

13(i)

This model is not a good fit for the data as the model gives the following:

P(less than 14 mins) = 0.31 = 31%

P(more than 18 mins) ≈ 0.07 = 7%

So the percentages are the wrong way round.

(ii) 2 marks

13(ii)

The mean that she should use is 17.

(iii) 0 marks

13(iii)

$H_0: \mu = 16$ for the Korea journey time in mins

$H_1: \mu \neq 16$ for the journey time in mins

Two tail test at the 5% level M0

Smallest possible value of n = 12.71

n is in days so 12.71 -> 12 days

Examiner commentary

Full marks were credited in part (i) for a correct response. Full marks were also credited in part (ii), despite the fact that there was no working to justify the answer.

Although this candidate has attempted to express some ideas for part (iii), there was insufficient progress to earn any credit here.

Question 14(i)(*A*), (i)(*B*),(ii), (iii), (iv) and (v)

14 The pre-release material includes data on unemployment rates in different countries. A sample from this material has been taken. All the countries in the sample are in Europe. The data have been grouped and are shown in Fig 14.1.

Unemployment rate	0–	5–	10–	15–	20–	35–50
Frequency	15	21	5	5	2	2

Fig. 14.1

A cumulative frequency curve has been generated for the sample data using a spreadsheet. This is shown in Fig. 14.2.

Fig. 14.2

Hodge used Fig. 14.2 to estimate the median unemployment rate in Europe. He obtained the answer 5.0. The correct value for this sample is 6.9.

(i) (*A*) There is a systematic error in the diagram.

- Identify this error.
- State how this error affects Hodge's estimate. **[2]**

(*B*) There is another factor which has affected Hodge's estimate.

- Identify this factor.
- State how this factor affects Hodge's estimate. **[2]**

(ii) Use your knowledge of the pre-release material to give another reason why any estimation of the median unemployment rate in Europe may be unreliable. **[1]**

(iii) Use your knowledge of the pre-release material to explain why it is very unlikely that the sample has been randomly selected from the pre-release material. **[1]**

The scatter diagram shown in Fig. 14.3 shows the unemployment rate and life expectancy at birth for the 47 countries in the sample for which this information is available.

Scatter diagram to show life expectancy at birth against unemployment rate

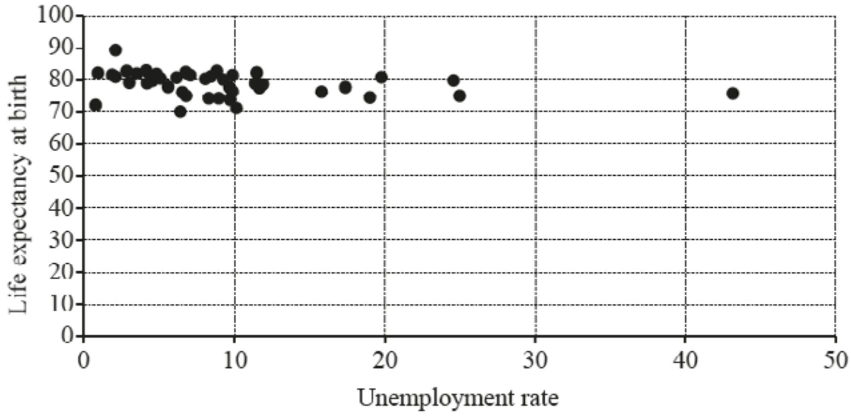


Fig. 14.3

The product moment correlation coefficient for the 47 items in the sample is – 0.2607.
The p-value associated with $r = -0.2607$ and $n = 47$ is 0.0383.

(iv) Does this information suggest that there is an association between unemployment rate and life expectancy at birth in countries in Europe? **[2]**

Hodge uses the spreadsheet tools to obtain the equation of a line of best fit for this data.

(v) The unemployment rate in Kosovo is 35.3, but there is no data available on life expectancy. Is it reasonable to use Hodge's line of best fit to estimate life expectancy at birth in Kosovo? **[1]**

Exemplar 1

(i)(*A*) 2 marks

14(i)(*A*)	Median = 25th
	Plotted the values in the middle of each group. Cumulative frequency graphs are plotted at the upper bounds of each group. His median is therefore lower than ~~expe~~ it should be.

(i)(*B*) 0 marks

~~Det Means his graph is close to~~
~~bunched lower~~
~~His plotted points~~
Decreases his estimate from what it should be.

(ii) 0 marks

14(ii) Some countries have a much higher GDP so more ~~people~~ can afford to ~~be a~~ employ companies. people. This means a higher GDP decrease unemployment.

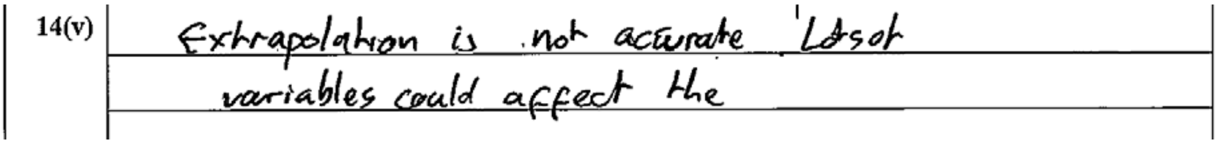
(iii) 1 mark

14(iii) All the countries happen to be from Europe but the data set had about 200 countries. The probability of only European countries selected is so small that it is unlikely to be random.

(iv) 2 marks

14(iv)
$H_0: \rho = 0$
$H_1: \rho \neq 0$
p= ~~0.2607~~ 0.0383 for r = -2.607
0.0383 < 0.05 so at 5% significance level this appears critical.
The ~~data~~ association shows a very weak negative association between life expectancy and unemployment rate.
However correlation does not equal causation as other factors also affect life expectancy.

(v) 0 marks


14(v) Extrapolation is not accurate Lots of variables could affect the

Examiner commentary

In part (i) (*A*), one mark was credited for observing that the cumulative frequencies were plotted at the mid-points of the intervals, instead of the ends of the intervals. The second mark was credited for noting that this results in a reduction in the estimate. The original intention of the mark scheme was that the second mark would only be credited to those candidates who noted that this should reduce the estimate by 2.5. The recognition of the mitigation of this effect due to grouping was tested in part (*B*). This candidate did not comment directly on the effect of grouping the data. Had the crossed out work contained anything creditworthy, marks would have been available because the replacement work is not complete. It is worth noting that no candidates managed to earn full marks on part (i).

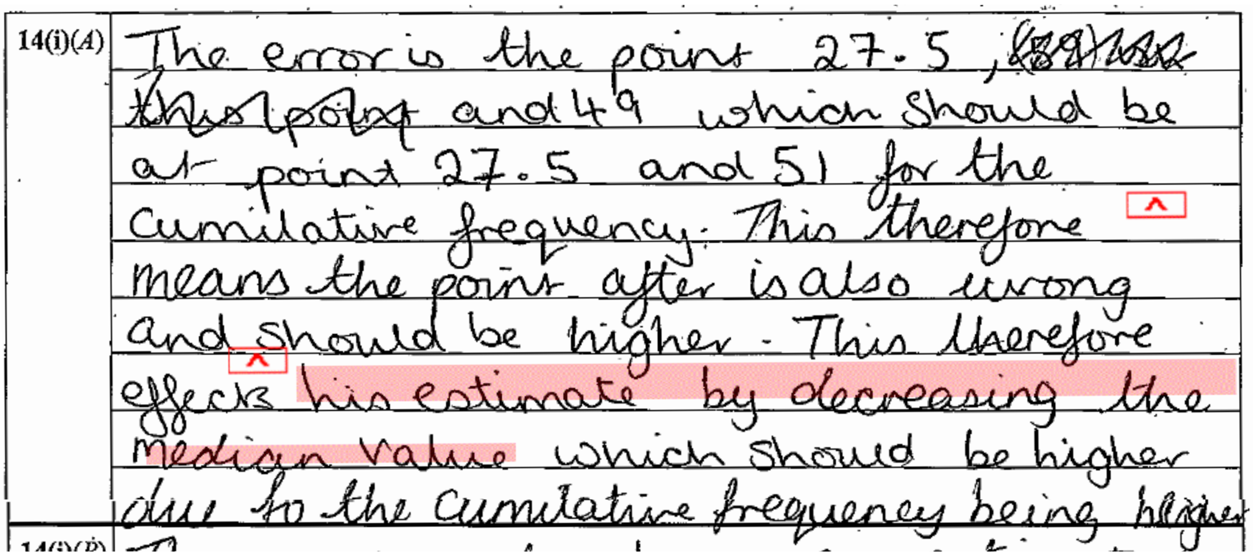
In parts (ii) and (iii), only those candidates who were familiar with the large data set earned the marks. Part (ii) focused on issues related to the estimation of median values from the large data set, rather than general geographical or economic ideas.

In part (iv), candidates were expected to comment that there appears to be negative correlation, and then to compare the *p*-value with a significance level of their choice to infer whether or not there is evidence to suggest the correlation is significant.

In part (v), the response needed to focus on the data, and not a generic statement on extrapolation.

Exemplar 2

(i)(*A*) 0 marks


14(i)(*A*) The error is the point 27.5, ~~this point~~ and 49 which should be at point 27.5 and 51 for the cumulative frequency. This therefore means the point after is also wrong and should be higher. This therefore effects his estimate by decreasing the median value which should be higher due to the cumulative frequency being higher

(i)(*B*) 1 mark

14(i)(*B*) The graph only shows an estimated value as the cumitative frequency was calculated using the midpoint of each unemployment rate group. It could be that the spread in each group is varied therefore giving a different answer. This effects his estimate as he is only given large groups and therefore his estimate will be less accurate.

BOD B1 B0

(ii) 0 marks

14(ii) Europe contains alot of diverse developed and undeveloped countries and therefore the estimations will be unreliable as there are bound to be alot of extremities on either side. This is a very small sample size

^

(iii) 0 marks

14(iii) Majority of the samples chosen lie within the 0 – 10 rate region which suggests alot more developed countres were chosen rather than a random more spread sample of both types

^

(iv) 0 marks

14(iv) No the information does not suggest there is an assoccation between unemployment and life expectancy because Life expectancy seems to be the same regardless of the unemployment rate. Life expectancy is within the 70 - 90 region for all countries

(v) 0 marks

14(v) No it is not. There must be a reason for the life expectancy for kosovo not being avaliable there could be esternal factors which effect it therefore using the line of best fit would be insufficien

Examiner commentary

In part (i) (*A*), only one midpoint was commented on, which was insufficient for the award of the first mark as an error with one point is not a systematic error. In part (*B*), the candidate conveyed understanding that grouping affected the answer, so B1 BOD was credited.

In common with many other candidates, this candidate made plausible comments, but they did not indicate knowledge of the large data set, so no credit was given.

In part (iv), there did not appear to be any attempt to interpret the scatter diagram and no marks were credited.

In part (v), the candidate again did not attempt to interpret the information given, so no mark was credited.

Question 15

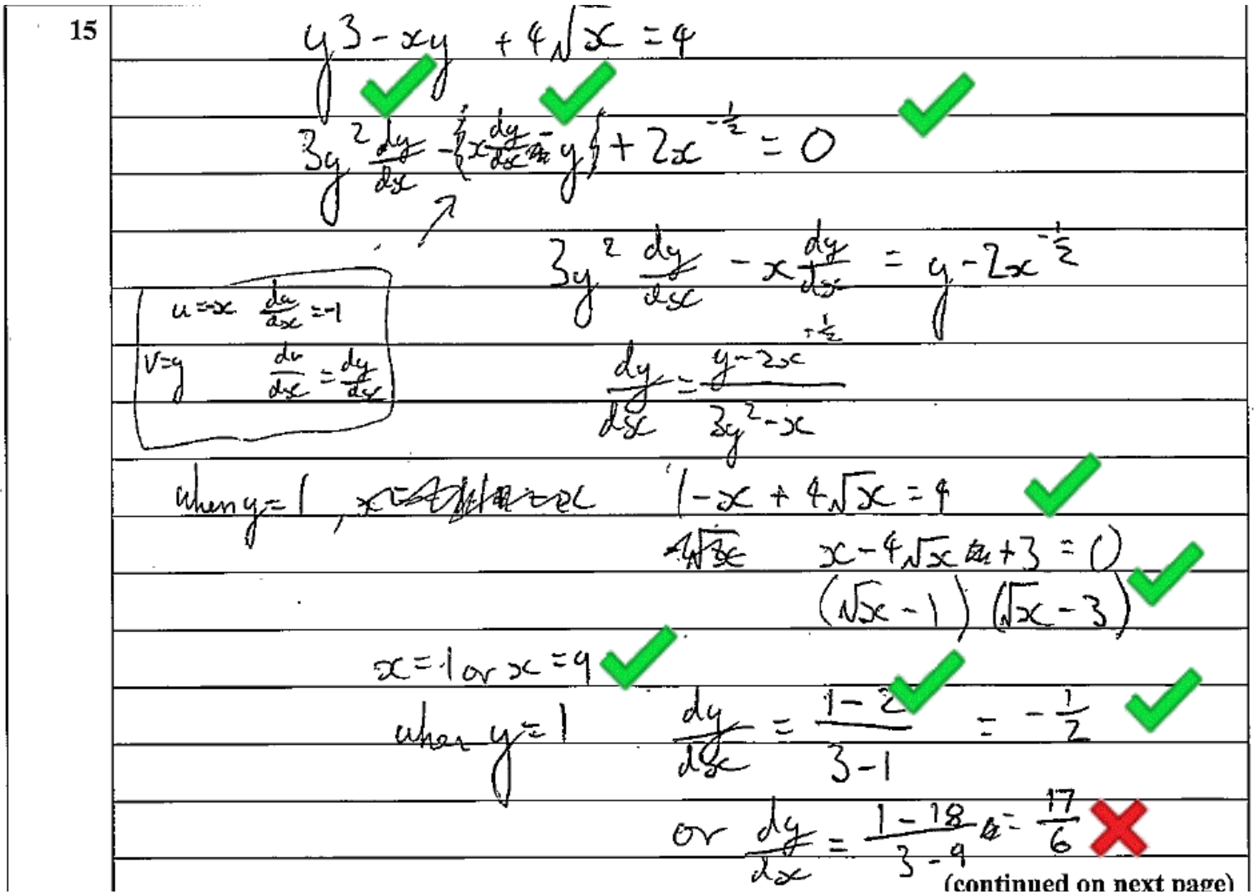
15 You must show detailed reasoning in this question.

The equation of a curve is

$$y^3 - xy + 4\sqrt{x} = 4 .$$

Find the gradient of the curve at each of the points where $y = 1$. **[9]**

Exemplar 1 — 8 marks


15

$y^3 - xy + 4\sqrt{x} = 4$

$3y^2 \frac{dy}{dx} - \{x\frac{dy}{dx} + y\} + 2x^{-\frac{1}{2}} = 0$

$u = x \quad \frac{du}{dx} = 1$

$v = y \quad \frac{dv}{dx} = \frac{dy}{dx}$

$3y^2 \frac{dy}{dx} - x\frac{dy}{dx} = y - 2x^{-\frac{1}{2}}$

$\frac{dy}{dx} = \frac{y - 2x^{+\frac{1}{2}}}{3y^2 - x}$

when $y = 1$, $1 - x + 4\sqrt{x} = 4$

$x - 4\sqrt{x} + 3 = 0$

$(\sqrt{x} - 1)(\sqrt{x} - 3)$

$x = 1$ or $x = 9$

when $y = 1$ $\frac{dy}{dx} = \frac{1-2}{3-1} = -\frac{1}{2}$

or $\frac{dy}{dx} = \frac{1-18}{3-9} = \frac{17}{6}$

(continued on next page)

Examiner commentary

This candidate's response is near perfect. However, it appears that with the final calculations the expression for $\frac{dy}{dx}$ has been misread. When the derivative is first simplified, the power of –½ is written on the line above. When the values of x and y are substituted in, it appears that the power of x in the numerator has been omitted, so we see – 18 in the numerator instead of – $^2/_3$ when the second gradient is calculated. We cannot be certain whether the first gradient follows from correct working or not, so this was credited BOD A1.

Exemplar 2

5 marks

15 $y^3 - xy + 4\sqrt{x} = 4$.

Points where $y = 1$,

✓

$1 - x + 4\sqrt{x} = 4$

$x - 4\sqrt{x} + 3 = 0$. ✓

$\sqrt{x} = y$

$y^2 - 4y + 3 = 0$.

$(y-3)(y-1) = 0$.

$\sqrt{x} = 3$ or $\sqrt{x} = 1$

$x = 9 \quad x = 1$

✓

$y^3 - xy + 4\sqrt{x} = 4$.

B1 M0

$\frac{dy}{dx} 3y^2 - x - \frac{dy}{dx} x + 2x^{-1/2} = 0$.

$\frac{dy}{dx}(3y^2 - x) = x - 2/\sqrt{x}$

$\frac{dy}{dx} = \frac{x - 2/\sqrt{x}}{3y^2 - x}$

$y = 1, x = 9$

M1

$\frac{dy}{dx} = \frac{9 - 2/3}{3 - 9} = \frac{-25}{18}$ ✗ at $(1, 9)$

$y = 1, x = 1$

✗

$\frac{dy}{dx} = \frac{1 - 2}{3 - 1} = -\frac{1}{2}$ at $(1, 1)$

Examiner commentary

The first three marks for finding the *x*-values are clearly earned, as is the B mark for correct use of the Chain Rule. Although the candidate attempted to use the Product rule to differentiate *xy*, the M1 was withheld because only one sign error was allowed for this mark. The subsequent A marks were all unavailable, although the M1 for substitution was credited because at least two terms were correct in the differentiated expression.

Exemplar 3 — 1 mark

15 $y^3 - xy + 4\sqrt{x} = 4$

$1^3 - x + 4\sqrt{x} = 4$

$1 - x + 4\sqrt{x} = 4$

$1 - x + 4(x^{1/2}) = 4$

$1 + 2x^{-1/2} = 0$

$1 - x + 4(x^{1/2}) = 4$

$\frac{dy}{dx} = -1 + 2x^{-1/2} = 0$

$2x^{-1/2} = 1$

$x^{-1/2} = 1/2$

$\frac{1}{\sqrt{x}} = \frac{1}{2}$

$x = 4$ $(4, 1)$ point on curve

15 (continued) $(4, 1)$: $y^3 - xy + 4\sqrt{x} = 4$

$y = mx + c$

$3y^2 - 1 + (\frac{1}{2}x^{-1/2}) = 4$

$3y^2 = 4 + 1 + \frac{1}{2}x^{-1/2}$

$= 5 + \frac{1}{2}x^{-1/2}$

$= 5 + \frac{1}{2} \times \frac{1}{\sqrt{2}}$

$3y^2 = 5 + \frac{1}{2\sqrt{x}}$

$y^2 = 3(5 + \frac{1}{2\sqrt{x}})$

$y^2 = 15 + \frac{3}{2\sqrt{x}}$

$y = \sqrt{15 + \dfrac{3}{2\sqrt{x}}}$

$y = \sqrt{15 + 3\left(2\left(x^{1/2}\right)\right)^{-1}}$

~~$y = 15 + 36$~~

$y = \left(15 + 6\left(x^{-1/2}\right)^{-1}\right)^{1/2}$

~~$y = \sqrt{15} +$~~

$y = \sqrt{15} + \sqrt{6}\ x^{1}$

$y = mx + c$

gradient $= \sqrt{6}$

✗

Examiner commentary

M1 was credited for the substitution of $y = 1$. Thereafter the attempt to solve the equation goes astray and no marks are earned. The candidate attempted differentiation of the expression following substitution of $y = 1$, which is not creditworthy. However, had the following attempt at differentiating implicitly been successful, the earlier part would have been overlooked in this case, as the attempt to differentiate implicitly would have been viewed as the last full attempt at this part of the question.

Question 16(i), (ii), (iii), (iv) and (v)

16 In the first year of a course, an A-level student, Aaishah, has a mathematics test each week. The night before each test she revises for t hours. Over the course of the year she realises that her percentage mark for a test, p, may be modelled by the following formula, where A, B and C are constants.

$$p - A \quad B(t \quad C)^2$$

- Aaishah finds that, however much she revises, her maximum mark is achieved when she does 2 hours revision. This maximum mark is 62.

- Aaishah had a mark of 22 when she didn't spend any time revising.

(i) Find the values of A, B and C. **[3]**

(ii) According to the model, if Aaishah revises for 45 minutes on the night before the test, what mark will she achieve? **[2]**

(iii) What is the maximum amount of time that Aaishah could have spent revising for the model to work? **[2]**

In an attempt to improve her marks Aaishah now works through problems for a total of t hours over the three nights before the test. After taking a number of tests, she proposes the following new formula for p.

$$p = 22 + 68(1 - \mathrm{e}^{\;0.8t})$$

For the next three tests she recorded the data in Fig. 16.

t	1	3	5
p	59	84	89

Fig. 16

(iv) Verify that the data is consistent with the new formula. **[2]**

(v) Aaishah's tutor advises her to spend a minimum of twelve hours working through problems in future. Determine whether or not this is good advice. **[2]**

Exemplar 1

(i) 3 marks

16(i)

~~When: $t=2$, $P=62$~~ ~~When $t=0$, $P=22$~~

~~$62 = A - B(2-C)^2$~~ $22 = A - BC^2$

Turning point: $(2, 62)$

$\Rightarrow C = 2,\ A = 62$

$22 = 62 - B(-2)^2$

$\frac{-40}{-4} = B = 10$ ✓

(ii) 2 marks

16(ii)

$P = 62 - 10(t-2)^2$

$t = \frac{3}{4}$ $\quad P = \frac{371}{8} = 46.375$ ✓

(iii) 2 marks

16(iii)

$0 = 62 - 10(t-2)^2$

$\frac{62}{10} = (t-2)^2$

$\frac{\pm\sqrt{155}}{5} + 2 = t$ ↖ no negative time

$\Rightarrow t = 2 + \frac{10 + \sqrt{155}}{5} = 4.49$ hours ✓

(iv) 2 marks

16(iv)

$t=1, \quad p = 22 + 68(1 - e^{-0.8}) \approx 59.4$

≈ 59

$t=3, \quad p = 83.8 \approx 84$

$t=5 \quad p = 88.75.. \approx 89$

(v) 2 marks

16(v)

No, because she can spend only 7 hours and get the same result

$22 + 68(1 - e^{-0.8 \times 7}) = 89.7 \approx 90$

$22 + 68(1 - e^{-0.8 \times 12}) = 89.9 \approx 90$

Examiner commentary

This candidate has provided a good clear response to each part, gaining full credit.

Exemplar 2

(i) 0 marks

16(i)

$62 = A - B(2-c)^2$

$22 = A - B(0-c)^2$

	2	$-c$
2	4	$-2c$
$-c$	$-2c$	c^2

~~$62 = A - B(c^2 - 4c + 4)$~~

~~$22 = A - B(c^2)$~~

$62 \div 22 = 2.818$

~~$c = 0.295$~~

$c = 0.321$

$A - B \approx 2.4$

$A = 30$

$B = 8$

$c = 0.321$

(ii) 1 mark

16(ii)

$P = A - B\left(\frac{3}{4} - C\right)^2$ M1

$P = 30 - 8\left(\frac{3}{4} - 0.321\right)^2$

P = ~~ZAZA~~ 40 marks

(iv) 1 mark

16(iv)

$59 = 22 + 68\left(1 - e^{-0.8}\right)$

$22 + 68\left(1 - e^{-0.8}\right) = 59.44 = 59$ as required SC

$84 = 22 + 68\left(1 - e^{-2.4}\right)$

$22 + 68\left(1 - e^{-2.4}\right) = 83.83 = 84$ as required

^ M0

(v) 0 marks

16(v)

$P = 22 + 68\left(1 - e^{-7.6}\right)$

$P = 89.97 \rightarrow 90$

This is good advice as it will improve her mark

Examiner commentary

In part (I), no marks were credited because all three constants were incorrect.

However, M1 was credited in part (ii) for substitution of $t = 0.75$ in their equation, follow through.

Part (iii) was not attempted.

There was a partial attempt at part (iv). However, M1 was not credited because only two values were found. These were found correctly, however, so SC1 was available.

In the final part the candidate has worked with $e^{-7.6}$, which follows from $t = 9.5$. Whilst the value calculated is correct, 9.5 is less than 12, so it could not reasonably be regarded as "large". It is possible that the candidate wrote $e^{-9.6}$ and then misread this as $e^{-7.6}$, but it is not possible to be sure. This is another instance in which the importance of clear writing cannot be over-emphasised.

Question 17(i) and (ii)

17 **(i)** Express $\dfrac{(x^2-8x+9)}{(x+1)(x-2)^2}$ in partial fractions. **[5]**

(ii) Express y in terms of x given that

$$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{y(x^2-8x+9)}{(x+1)(x-2)^2} \text{ and } y=16 \text{ when } x=3.$$

[7]

Exemplar 1

(i) 5 marks

17(i)

$$\frac{(x^2-8x+9)}{(x+1)(x-2)^2} \equiv \frac{A}{x+1}+\frac{B}{x-2}+\frac{C}{(x-2)^2}$$ ✓

$A(x-2)^2+B(x+1)(x-2)+C(x+1)=x^2-8x+9$ ✓

$x=2:\quad 3C=2^2-8(2)+9=-3$

$C=-1$

$x=-1:\quad 9(-3)^2A=18 \therefore 9A=18$

$A=2$

Equate coefficients: $2x^2+Bx^2=x^2$

$B=1-2=-1$

so partial fractions are:

$$\frac{2}{x+1}-\frac{1}{x-2}-\frac{1}{(x-2)^2}$$ ✓

(ii) 7 marks

17(ii)

$$\frac{dy}{dx} = \frac{y(x^2 - 8x + 9)}{(x+1)(x-2)^2}$$

$$\int \frac{1}{y}\,dy = \int \frac{x^2 - 8x + 9}{(x+1)(x-2)^2}\,dx$$

$$\ln y = \int \frac{2}{x+1} - \frac{1}{x-2} - \frac{1}{(x-2)^2}\,dx$$

$$\int \frac{2}{x+1}\,dx = 2\ln|x+1|$$

$$\int \frac{-1}{(x-2)^2}\,dx \quad : \quad u = x-2 \quad \frac{du}{dx} = 1 \quad dx = du$$

$$\int \frac{-1}{x-2}\,dx = -\ln|x-2|$$

$$\int -\frac{1}{u^2}\,du = \int -u^{-2}\,du$$

$$= u^{-1} = \frac{1}{x-2} + c$$

$$\ln y = 2\ln|x+1| - \ln|x-2| + \frac{1}{x-2} + c$$

$$\ln y = \ln\left|\frac{(x+1)^2}{x-2}\right| + \frac{1}{x-2} + c \qquad y = 16,\ x = 3$$

$$\ln 16 = \ln\left(\frac{4^2}{1}\right) + \frac{1}{1} + c$$

$$c = \ln 16 - \ln 4^2 - 1 = -1$$

$$\ln y = \ln\left|\frac{(x+1)^2}{x-2}\right| + \frac{1}{x-2} - 1$$

$$y = e^{\left(\ln\left(\frac{(x+1)^2}{x-2}\right) + \frac{1}{x-2} - 1\right)}$$

$$y = \frac{(x+1)^2}{x-2}\,e^{(x-2)^{-1} - 1}$$

Examiner commentary

In part (i), the work is clearly laid out and full marks were credited.

In part (ii), it is worth noting that the final answer is different to the one quoted in the mark scheme. Full marks were credited because the scheme has "oe" next to the final answer – an abbreviation for "or equivalent", so any correct form would have earned the marks.

Exemplar 2

(i) 5 marks

17(i)

$$\frac{x^2-8x+9}{(x+1)(x-2)^2}$$

$$x^2-8x+9 = A(x-2)^2 + B(x+1)(x-2) + C(x+1)$$

$x=2,\quad -3 = 3C \Rightarrow C=-1$

$x=-1,\quad 18 = 9A \Rightarrow A = 2$

$x=0,\quad 9 = 4A - 2B + C$

$9 = 8 - 2B - 1$

$9 = 7 - 2B \therefore B = -1$

$$\frac{(x^2-8x+9)}{(x+1)(x-2)^2} = \frac{2}{x+1} - \frac{1}{x-2} - \frac{1}{(x-2)^2}$$

(ii) 3 marks

17(ii)

$$\frac{dy}{dx} = \frac{y(x^2-8x+9)}{(x+1)(x-2)^2}$$

$$\frac{1}{y}\frac{dy}{dx} = \frac{2}{x+1} - \frac{1}{x-2} - \frac{1}{(x-2)^2}$$

$$\int \frac{1}{y}\,dy = \int\left(\frac{2}{x+1} - \frac{1}{x-2} - \frac{1}{(x-2)^2}\right)dx$$

u^{-2}

$-\frac{1}{u}$

$$\ln y = 2\ln|x+1| - \ln|x-2| - \frac{1}{x-2} + c$$

$$\ln y = \ln\left|\frac{(x+1)^2}{x-2}\right| - \frac{1}{x-2} + c$$

$$y = \frac{(x+1)^2}{x-2} - Ae^{-\frac{1}{x-2}}$$

Examiner commentary

Part (i) is fully correct so full marks were credited.

In part (ii), the first two method marks were credited for the separation of variables and the use of partial fractions from part (i). The first A mark was credited for a correct natural log integral on the RHS. However, there was a sign error in the third term, so the second A mark was withheld. There was no attempt to substitute the values of x and y, and the exponentiation of each side was incorrect, so no further marks were credited.

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Qualification and notional component raw mark grade boundaries June 2018 series

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New AS Levels

AS GCE Mathematics B (MEI)										
				Max Mark	a	b	c	d	e	u
H630	01	Pure Mathematics and Mechanics	Raw	70	44	38	33	28	23	0
H630	02	Pure Mathematics and Statistics	Raw	70	50	45	39	33	28	0
			Overall	140	94	83	72	61	51	0

AS GCE Further Mathematics B (MEI) (H635)										
				Max Mark	a	b	c	d	e	u
Y410	01	Core Pure	Raw	60	46	41	36	32	28	0
Y411	01	Mechanics a	Raw	60	37	32	27	22	18	0
Y412	01	Statistics a	Raw	60	42	38	34	30	26	0
Y413	01	Modelling with Algorithms	Raw	60	37	33	29	25	22	0
Y414	01	Numerical Methods	Raw	60	35	29	24	19	14	0
Y415	01	Mechanics b	Raw		No entry in June 2018					
Y416	01	Statistics b	Raw	60	43	38	33	28	24	0
H635		*Option Y410+Y411+Y412*	Overall	180	125	111	98	85	72	0
H635		*Option Y410+Y411+Y413*	Overall	180	120	107	94	81	68	0
H635		*Option Y410+Y411+Y414*	Overall	180	118	103	88	74	60	0
H635		*Option Y410+Y412+Y413*	Overall	180	125	112	100	88	76	0
H635		*Option Y410+Y412+Y414*	Overall	180	123	109	95	81	68	0
H635		*Option Y410+Y412+Y416*	Overall	180	131	117	104	91	78	0
H635		*Option Y410+Y413+Y414*	Overall	180	118	104	90	77	64	0

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New A Levels

A Level Mathematics B (MEI)

				Max Mark	a*	a	b	c	d	e	u
H640	01	Pure Mathematics and Mechanics	Raw	100	81	74	67	59	52	45	0
H640	02	Pure Mathematics and Statistics	Raw	100	75	68	61	54	47	40	0
H640	03	Pure Mathematics and Comprehension	Raw	75	62	55	48	42	36	30	0
			Overall	275	218	197	176	155	135	115	0