Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics
Question Paper

Wednesday 6 June 2018 – Morning
Time allowed: 2 hours

You must have:
- Printed Answer Booklet

You may use:
- a scientific or graphical calculator

7020513453

INSTRUCTIONS
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Complete the boxes provided on the Printed Answer Booklet with your name, centre number and candidate number.
- Answer **all** the questions.
- **Write your answer to each question in the space provided in the Printed Answer Booklet.** Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.
- The acceleration due to gravity is denoted by $g\,\mathrm{m\,s^{-2}}$. Unless otherwise instructed, when a numerical value is needed, use $g = 9.8$.

INFORMATION
- The total number of marks for this paper is **100**.
- The marks for each question are shown in brackets **[ ]**.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is used. You should communicate your method with correct reasoning.
- The Printed Answer Booklet consists of **20** pages. The Question Paper consists of **12** pages.

Formulae A Level Mathematics B (MEI) (H640)

Arithmetic series

$S_n = \frac{1}{2}n(a+l) = \frac{1}{2}n\{2a+(n-1)d\}$

Geometric series

$S_n = \dfrac{a(1-r^n)}{1-r}$

$S_\infty = \dfrac{a}{1-r}$ for $|r| < 1$

Binomial series

$(a+b)^n = a^n + {}^n\mathrm{C}_1 a^{n-1}b + {}^n\mathrm{C}_2 a^{n-2}b^2 + \ldots + {}^n\mathrm{C}_r a^{n-r}b^r + \ldots + b^n \qquad (n \in \mathbb{N}),$

where ${}^n\mathrm{C}_r = {}_n\mathrm{C}_r = \dbinom{n}{r} = \dfrac{n!}{r!(n-r)!}$

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2!}x^2 + \ldots + \dfrac{n(n-1)\ldots(n-r+1)}{r!}x^r + \ldots \qquad (|x| < 1,\ n \in \mathbb{R})$

Differentiation

$\mathrm{f}(x)$	$\mathrm{f}'(x)$
$\tan kx$	$k\sec^2 kx$
$\sec x$	$\sec x \tan x$
$\cot x$	$-\mathrm{cosec}^2 x$
$\mathrm{cosec}\, x$	$-\mathrm{cosec}\, x \cot x$

Quotient Rule $y = \dfrac{u}{v},\ \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{v\dfrac{\mathrm{d}u}{\mathrm{d}x} - u\dfrac{\mathrm{d}v}{\mathrm{d}x}}{v^2}$

Differentiation from first principles

$\mathrm{f}'(x) = \lim\limits_{h\to 0}\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}$

Integration

$\displaystyle\int \frac{\mathrm{f}'(x)}{\mathrm{f}(x)}\,\mathrm{d}x = \ln|\mathrm{f}(x)| + c$

$\displaystyle\int \mathrm{f}'(x)(\mathrm{f}(x))^n\,\mathrm{d}x = \frac{1}{n+1}(\mathrm{f}(x))^{n+1} + c$

Integration by parts $\displaystyle\int u\frac{\mathrm{d}v}{\mathrm{d}x}\,\mathrm{d}x = uv - \int v\frac{\mathrm{d}u}{\mathrm{d}x}\,\mathrm{d}x$

Small angle approximations

$\sin\theta \approx \theta$, $\cos\theta \approx 1-\frac{1}{2}\theta^2$, $\tan\theta \approx \theta$ where θ is measured in radians

Trigonometric identities

$\sin(A \pm B) = \sin A\cos B \pm \cos A \sin B$

$\cos(A \pm B) = \cos A\cos B \mp \sin A \sin B$

$\tan(A \pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ $\qquad \left(A \pm B \neq \left(k+\frac{1}{2}\right)\pi\right)$

Numerical methods

Trapezium rule: $\displaystyle\int_a^b y\,\mathrm{d}x \approx \frac{1}{2}h\{(y_0+y_n)+2(y_1+y_2+\ldots+y_{n-1})\}$, where $h = \dfrac{b-a}{n}$

The Newton-Raphson iteration for solving $\mathrm{f}(x)=0$: $x_{n+1} = x_n - \dfrac{\mathrm{f}(x_n)}{\mathrm{f}'(x_n)}$

Probability

$\mathrm{P}(A \cup B) = \mathrm{P}(A) + \mathrm{P}(B) - \mathrm{P}(A \cap B)$

$\mathrm{P}(A \cap B) = \mathrm{P}(A)\mathrm{P}(B \mid A) = \mathrm{P}(B)\mathrm{P}(A \mid B)$ **or** $\mathrm{P}(A \mid B) = \dfrac{\mathrm{P}(A \cap B)}{\mathrm{P}(B)}$

Sample variance

$s^2 = \dfrac{1}{n-1}S_{xx}$ where $S_{xx} = \Sigma(x_i-\bar{x})^2 = \Sigma x_i^2 - \dfrac{(\Sigma x_i)^2}{n} = \Sigma x_i^2 - n\bar{x}^2$

Standard deviation, $s = \sqrt{\text{variance}}$

The binomial distribution

If $X \sim \mathrm{B}(n, p)$ then $\mathrm{P}(X = r) = {}^n\mathrm{C}_r p^r q^{n-r}$ where $q = 1-p$

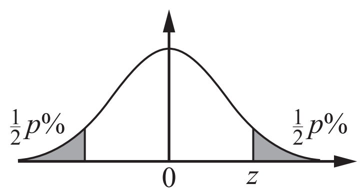
Mean of X is np

Hypothesis testing for the mean of a Normal distribution

If $X \sim \mathrm{N}(\mu, \sigma^2)$ then $\bar{X} \sim \mathrm{N}\left(\mu, \dfrac{\sigma^2}{n}\right)$ and $\dfrac{\bar{X}-\mu}{\sigma/\sqrt{n}} \sim \mathrm{N}(0,1)$

Percentage points of the Normal distribution

p	10	5	2	1
z	1.645	1.960	2.326	2.576



Kinematics

Motion in a straight line

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = \frac{1}{2}(u+v)t$

$v^2 = u^2 + 2as$

$s = vt - \frac{1}{2}at^2$

Motion in two dimensions

$\mathbf{v} = \mathbf{u} + \mathbf{a}t$

$\mathbf{s} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$

$\mathbf{s} = \frac{1}{2}(\mathbf{u}+\mathbf{v})t$

$\mathbf{s} = \mathbf{v}t - \frac{1}{2}\mathbf{a}t^2$

Answer **all** the questions

Section A (23 marks)

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. **[3]**

2 By considering a change of sign, show that the equation $e^x-5x^3=0$ has a root between 0 and 1. **[2]**

3 **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta+2\tan\theta=4$ for $0° \leqslant \theta < 360°$. **[4]**

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\mathrm{m\,s^{-1}}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. **[2]**

(ii) Find the magnitude of the force that Rory applies. **[2]**

5 The position vector **r** metres of a particle at time *t* seconds is given by

$$\mathbf{r}=(1+12t-2t^2)\mathbf{i}+(t^2-6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time *t*. **[2]**

(ii) Determine whether the particle is ever stationary. **[2]**

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. **[2]**

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

(iii) Determine whether Baraka saves more in two years than Aleela. **[3]**

Answer **all** the questions

Section B (77 marks)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

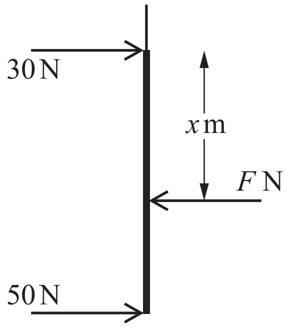


Fig. 7

(i) Find the value of F. **[1]**

(ii) Find the value of x. **[2]**

8 **(i)** Show that $8\sin^2 x\cos^2 x$ can be written as $1-\cos 4x$. **[3]**

(ii) Hence find $\int \sin^2 x\cos^2 x\,\mathrm{d}x$. **[3]**

9 A pebble is thrown horizontally at $14\,\mathrm{m\,s^{-1}}$ from a window which is $5\,\mathrm{m}$ above horizontal ground. The pebble goes over a fence $2\,\mathrm{m}$ high $d\,\mathrm{m}$ away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

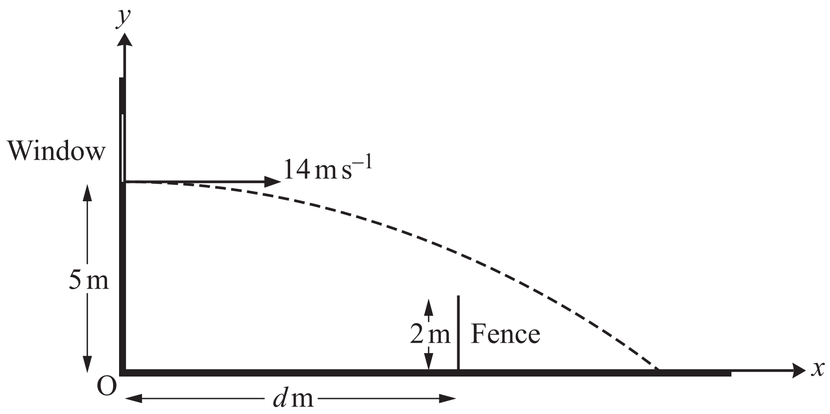


Fig. 9

(i) Find the time the pebble takes to reach the ground. **[3]**

(ii) Find the cartesian equation of the trajectory of the pebble. **[4]**

(iii) Find the range of possible values for d. **[3]**

10 Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

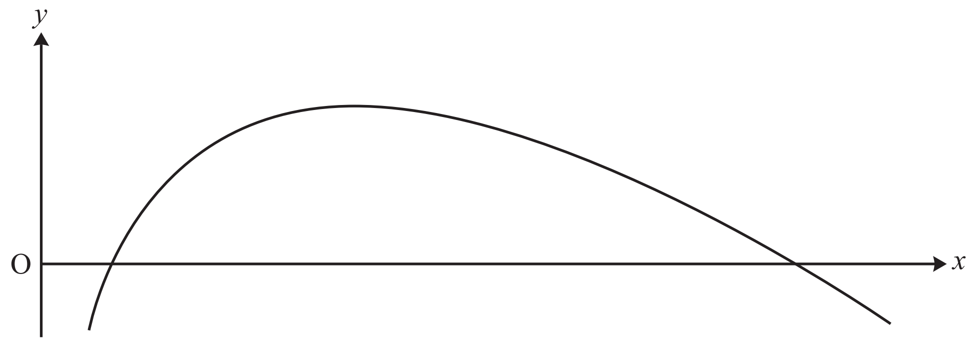


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

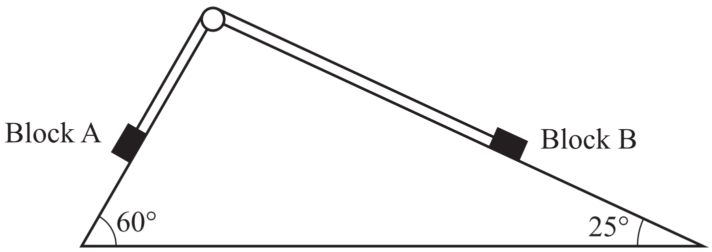


Fig. 11

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

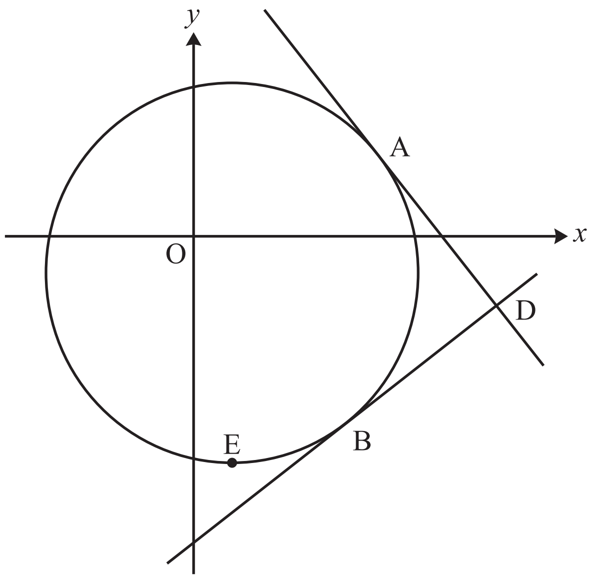


Fig. 12

(i) Write down the coordinates of C, the centre of the circle. **[1]**

(ii) (*A*) Show that the line $4y=3x-32$ is a tangent to the circle. **[4]**

(*B*) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. **[1]**

(iii) Prove that ADBC is a square. **[3]**

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

13 The function $\mathrm{f}(x)$ is defined by $\mathrm{f}(x) = \sqrt[3]{27-8x^3}$. Jenny uses her scientific calculator to create a table of values for $\mathrm{f}(x)$ and $\mathrm{f}'(x)$.

x	$\mathrm{f}(x)$	$\mathrm{f}'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $\mathrm{f}'(x)$ and hence explain why the calculator gives an error for $\mathrm{f}'(1.5)$. **[3]**

(ii) Find the first three terms of the binomial expansion of $\mathrm{f}(x)$. **[3]**

(iii) Jenny integrates the first three terms of the binomial expansion of $\mathrm{f}(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. **[3]**

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,\mathrm{d}x$. The graph of $y = \mathrm{f}(x)$ is shown in Fig. 13.

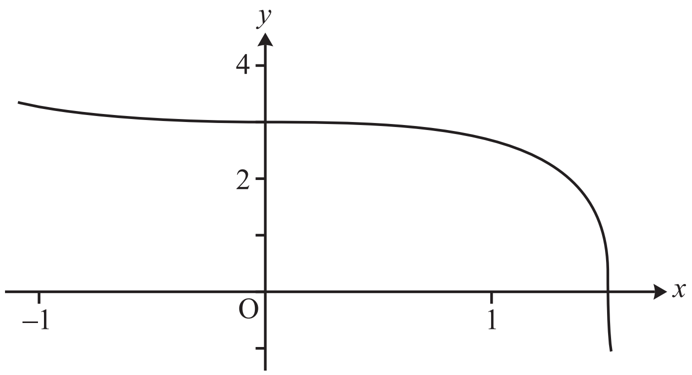


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

14 The velocity of a car, $v\,\mathrm{m\,s^{-1}}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\mathrm{m\,s^{-1}}$ and it accelerates to $11.4\,\mathrm{m\,s^{-1}}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

(ii) Explain why this model is not appropriate in the long term. **[1]**

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\mathrm{m\,s^{-1}}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

END OF QUESTION PAPER

Oxford Cambridge and RSA

A Level Mathematics B (MEI)

H640/01 Pure Mathematics and Mechanics

Printed Answer Booklet

Wednesday 6 June 2018 – Morning

Time allowed: 2 hours

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- Question Paper H640/01 (inserted)

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Last name	
Centre number	
Candidate number	

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Section A (23 marks)

1	
2	

3	
4(i)	
4(ii)	

5(i)	
5(ii)	

6(i)

6(ii)

6(iii)	

Section B (77 marks)

7(i)	
7(ii)	

8(i)	
8(ii)	

9(i)	
9(ii)	

9(iii)	
10	
	(answer space continued on next page)

10	(continued)

11(i)	
11(ii)	

12(i)	
12(ii)(*A*)	
12(ii)(*B*)	

12(iii)	
12(iv)	
	(answer space continued on next page)

12(iv)	(continued)
13(i)	

13(ii)	
13(iii)	

13(iv)	
13(v)	

14(i)	
14(ii)	
14(iii)	

14(iv)	
14(v)	

14(vi)	
14(vii)	

GCE

Mathematics B (MEI)

Unit **H640/01**: Pure Mathematics and Mechanics

Advanced Subsidiary GCE

Mark Scheme for June 2018

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, Cambridge Nationals, Cambridge Technicals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining a result or establishing a given result
dep*	Mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working
AG	Answer given
awrt	Anything which rounds to
BC	By Calculator
DR	This indicates that the instruction **In this question you must show detailed reasoning** appears in the question.

Subject-specific Marking Instructions for A Level Mathematics B (MEI)

a Annotations should be used whenever appropriate during your marking. The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded. For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct solutions leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly. Correct but unfamiliar or unexpected methods are often signalled by a correct result following an apparently incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
If you are in any doubt whatsoever you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, e.g. by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, e.g. wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more ‘method’ steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation ‘dep*’ is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation FT implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only – differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, what is acceptable will be detailed in the mark scheme. If this is not the case please, escalate the question to your Team Leader who will decide on a course of action with the Principal Examiner.
Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be ‘follow through’. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Unless units are specifically requested, there is no penalty for wrong or missing units as long as the answer is numerically correct and expressed either in SI or in the units of the question. (e.g. lengths will be assumed to be in metres unless in a particular question all the lengths are in km, when this would be assumed to be the unspecified unit.) We are usually quite flexible about the accuracy to which the final answer is expressed; over-specification is usually only penalised where the scheme explicitly says so. When a value is given in the paper only accept an answer correct to at least as many significant figures as the given value. This rule should be applied to each case. When a value is not given in the paper accept any answer that agrees with the correct value to 2 s.f. Follow through should be used so that only one mark is lost for each distinct accuracy error, except for errors due to premature approximation which should be penalised only once in the examination. There is no penalty for using a wrong value for g. E marks will be lost except when results agree to the accuracy required in the question.

g Rules for replaced work: if a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests; if there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others. NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a genuine misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate’s data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question. Marks designated as cao may be awarded as long as there are no other errors. E marks are lost unless, by chance, the given results are established by equivalent working. ‘Fresh starts’ will not affect an earlier decision about a misread. Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

i If a graphical calculator is used, some answers may be obtained with little or no working visible. Allow full marks for correct answers (provided, of course, that there is nothing in the wording of the question specifying that analytical methods are required). Where an answer is wrong but there is some evidence of method, allow appropriate method marks. Wrong answers with no supporting method score zero. If in doubt, consult your Team Leader.

j If in any case the scheme operates with considerable unfairness consult your Team Leader.

Question			Answer	Marks	AOs		Guidance
1			**EITHER** $f(2)=3\times 2^3-8\times 2^2+3\times 2+2=24-32+6+2=0$ Therefore by the factor theorem $(x-2)$ is a factor	 **M1** **A1** **E1** **[3]**	 **1.1a** **1.1b** **2.2a**	**AG** Function notation need not be used Zero must be seen Reason required	
			OR $f(x)=(x-2)(3x^2-2x-1)$ No remainder so $(x-2)$ is a factor	 **M1** **A1** **E1** **[3]**		 Using algebraic division as far as $3x^2$ Correct quotient Reason required	
2			When $x=0$ $e^0-5\times 0^3=1>0$ When $x=1$ $e^1-5\times 1^3=e-5<0$ So [as the function is continuous and there is a change of sign] there is a root between 0 and 1	**M1** **E1** **[2]**	**1.1a** **2.2a**	Attempting to evaluate the function at both values Conclusion from correct values	
3			$(1+\tan^2\theta)+2\tan\theta=4$ $\tan^2\theta+2\tan\theta-3=0$ $(\tan\theta-1)(\tan\theta+3)=0$ When $\tan\theta=1$, $\theta=45°, 225°$ When $\tan\theta=-3$, $\theta=108.4°, 288.4°$	 **M1** **M1** **A1** **A1** **[4]**	 **3.1a** **1.1a** **1.1b** **1.1b**	**DR** Using appropriate trig identity Showing algebraic method for solving their quadratic Any two correct values for θ All correct values for θ and no extras in the interval. Ignore values outside the required interval.	Must attempt to reach an equation with only one trig function eg $20\cos^4\theta-12\cos^2\theta+1=0$ Or $\sqrt{5}\sin(2\theta-63.4°)=1$
4	(i)		$v^2=u^2+2as$ $1.2^2=2\times a\times 1.8$ $a=0.4$ m s^{-2}	 **M1** **A1** **[2]**	 **3.3** **1.1b**	 Using suitable suvat equation(s) leading to value for a	

Question			Answer	Marks	AOs		Guidance
	(ii)		$F-19=2.8a$ $F=20.12$ (20.1 N to 3sf)	**M1** **A1** **[2]**	**3.3** **1.1b**	Using Newton's second law. All terms present. Allow 20 N FT their *a*	
5	**(i)**		$\mathbf{v}=\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t}=(12-2\times 2t)\mathbf{i}+(2t-6)\mathbf{j}$	**M1** **A1** **[2]**	**1.1a** **2.5**	Attempt to differentiate at least one coefficient Must use vector notation	
	(ii)		When $t=3$ both components of velocity are zero, so the particle is stationary at $t=3$.	**M1** **E1** **[2]**	**3.1a** **2.2a**	Equating at least one component of their vector velocity to zero Must be argued from two zero components	Do not allow M1 for solving $12-4t=2t-6$ unless at least one zero subsequently established

Question			Answer	Marks	AOs		Guidance
6	(i)		Arithmetic sequence with $a=50,\ d=20$ $S_{24}=\frac{24}{2}\left(2\times50+(24-1)20\right)$ = £6720	M1 A1 [2]	1.1a 1.1b	Using appropriate formula for sum of an arithmetic sequence with $a=50,\ d=20$ Allow full credit for any correct method	Allow for total written out in full
	(ii)		Each month is 12% more than the previous, so multiplied by 1.12 giving a geometric sequence with $a=50,\ r=1.12$	E1 [1]	2.4	Clear argument must include the value 1.12	
	(iii)		Geometric sequence with $a=50,\ r=1.12$ $S_{24}=\frac{50\left(1.12^{24}-1\right)}{0.12}$ = £5907.76 which is less than Aleela	M1 A1 E1 [3]	3.1a 1.1b 2.1	Using appropriate formula for sum of a geometric sequence with $a=50,\ r=1.12$ Allow any suitable rounding FT their values (dep on earning the M marks in part (i) and (iii))	Allow for total written out in full
7	(i)		$F=30+50=80$ N	B1 [1]	1.1a	Cao	
	(ii)		Taking moments about the top of the rod $Fx=50\times2$ $x=1.25$ m	M1 A1 [2]	3.3 1.1b	Or any other suitable point Cao	All necessary terms must be present. Each term must be a product of a force and a length.

Question			Answer	Marks	AOs		Guidance
8	(i)		**EITHER** $8\sin^2 x\cos^2 x = 2(1-\cos 2x)(1+\cos 2x)$ $= 2(1-\cos^2 2x) = 2-(1+2\cos 4x)$ $= 1-\cos 4x$	**M1** **M1** **E1** **[3]**	**3.1a** **3.1a** **2.1**	**AG** Using a double angle formula Second use of a double angle formula Clearly shown	
	(i)		**OR** $8\sin^2 x\cos^2 x = 2(2\sin x\cos x)^2$ $= 2\sin^2 2x$ $[= 1-\cos 2(2x)] = 1-\cos 4x$	**M1** **M1** **E1** **[3]**		Using a double angle formula Another use of a double angle formula Clearly shown	Allow any other valid sequence of identities used.
	(i)		**OR** $1-\cos 4x = 1-(1-2\sin^2 2x)$ $= 2\sin^2 2x$ $= 2(2\sin x\cos x)^2$ $= 8\sin^2 x\cos^2 x$	**M1** **M1** **E1** **[3]**		Using a double angle formula Another use of a double angle formula Clearly shown	
	(ii)		$\int \sin^2 x\cos^2 x \, dx = \frac{1}{8}\int 1-\cos 4x \, dx$ $= \frac{1}{8}x - \frac{1}{32}\sin 4x + c$	**M1** **A1** **A1** **[3]**	**1.1a** **1.1b** **1.1b**	Attempt to integrate both terms $\frac{1}{4}\sin 4x$ seen or implied All correct. Must include $+c$	

Question			Answer	Marks	AOs		Guidance
9	(i)		Vertical motion $u=0$ $s=ut+\frac{1}{2}at^2$ $-5=0-\frac{9.8}{2}t^2$ $t=\sqrt{\frac{10}{9.8}}=1.01$ s	**B1** **M1** **A1** **[3]**	**3.3** **3.4** **1.1b**	Using $u=0$ in the vertical direction to model horizontal motion soi Using suvat equation(s) to find t. Allow sign errors and incorrect value for u. Must follow from working where the signs are consistent.	
	(ii)		$x=14t$ $y=5-4.9t^2$ So cartesian equation is $y=5-4.9\left(\frac{x}{14}\right)^2\left[=5-\frac{x^2}{40}\right]$	**B1** **B1** **M1** **A1** **[4]**	**3.3** **3.3** **1.1a** **1.1b**	May be implied May be implied Attempt to eliminate t Any form	
	(iii)		EITHER When $y=2$ $y=5-\frac{x^2}{40}=2$ m $\frac{x^2}{40}=3$ $x=\sqrt{120}=10.9544...$ $[0<]d<11.0$ m	**M1** **A1** **E1** **[3]**	**3.4** **1.1b** **3.2a**	Using their equation of trajectory and $y=2$ Must be 11.0 or better Allow "Fence must be less than 10.95 m from the origin." FT their value	**SC2** for $d<\sqrt{80}\ [=8.94]$ **SC1** for $d=\sqrt{80}\ [=8.94]$
	(iii)		OR When $y=2$ $2=5-4.9t^2$ $t=0.782$ When $t=0.782$ $x=14\times0.782=10.95$ $[0<]d<11.0$ m	**M1** **A1** **A1** **[3]**		Both steps required for M1 Must be 11.0 or better Allow "Fence must be less than 10.95 m from the origin."	Allow if the origin is taken to be at window height and the top of the wall is 3m below the window. Signs must be consistent for A1

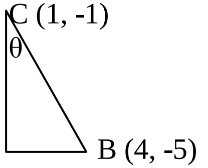
Question			Answer	Marks	AOs		Guidance
10			Curve crosses the x-axis when $y=0$ $y=(k-x)\ln x=0$	M1	3.1a	Attempt to solve $y=0$	
			Either $k-x=0$ or $\ln x=0$ $x=k$ or 1	A1	1.1b	Both roots required	
			EITHER Area = $\int_1^k (k-x)\ln x\ \mathrm{d}x$ Let $u=\ln x$, $\frac{\mathrm{d}v}{\mathrm{d}x}=k-x$, $\frac{\mathrm{d}u}{\mathrm{d}x}=\frac{1}{x}$, $v=kx-\frac{1}{2}x^2$	M1	2.1	Using integration by parts with $u=\ln x$, $\frac{\mathrm{d}v}{\mathrm{d}x}=k-x$ clearly argued	
			Area = $\left[\left(kx-\frac{1}{2}x^2\right)\ln x\right]_1^k-\int_1^k \frac{1}{x}\left(kx-\frac{1}{2}x^2\right)\mathrm{d}x$	A1	1.1b	Allow without limits	
			$\left[\left(kx-\frac{1}{2}x^2\right)\ln x\right]_1^k-\int_1^k \left(k-\frac{1}{2}x\right)\mathrm{d}x$	M1	3.1a	Simplifying the integrand	
			$\left[\left(kx-\frac{1}{2}x^2\right)\ln x-\left(kx-\frac{1}{4}x^2\right)\right]_1^k$	A1	1.1b	Second part correct	
			$\left(\left(k^2-\frac{1}{2}k^2\right)\ln k-\left(k^2-\frac{1}{4}k^2\right)\right)-\left(\left(k-\frac{1}{2}\right)\ln 1-\left(k-\frac{1}{4}\right)\right)$	M1dep	1.1a	Using limits. Dependendent on M mark for integration by parts	
			$=\frac{1}{2}k^2\ln k-\frac{3}{4}k^2+k-\frac{1}{4}$	A1 [8]	1.1b	Cao	

Question			Answer	Marks	AOs		Guidance
10			OR Integral split into two separate integrals $\int_1^k k\ln x \, dx$				
			Let $u=\ln x, \quad \frac{dv}{dx}=k, \quad \frac{du}{dx}=\frac{1}{x}, \quad v=kx$	M1		Using integration by parts with $u=\ln x, \; \frac{dv}{dx}=k$ or $u=\ln x, \; \frac{dv}{dx}=\pm x$ clearly argued	
			$=\left[kx\ln x\right]_1^k-\int_1^k \frac{1}{x}kx \, dx$	M1		Simplifying the integrand	
			$\left[kx\ln x\right]_1^k-\int_1^k k \, dx=\left[kx\ln x-kx\right]_1^k$ $\left(k^2\ln k-k^2\right)-\left(k\ln 1-k\right)=k^2\ln k-k^2+k$	M1dep		Substituition of limits seen in at least one integral. Dependendent on M mark for integration by parts	
			And Area = $\int_1^k x\ln x \, dx$ Let $u=\ln x, \quad \frac{dv}{dx}=x, \quad \frac{du}{dx}=\frac{1}{x}, \quad v=\frac{1}{2}x^2$				
			$=\left[\frac{1}{2}x^2\ln x\right]_1^k-\int_1^k \frac{1}{x}\times \frac{1}{2}x^2 dx$	A1		Both integrals correct at this stage Allow without limits	
			$\left[\frac{1}{2}x^2\ln x\right]_1^k-\int_1^k \frac{1}{2}x \, dx$ $\left[\frac{1}{2}x^2\ln x-\frac{1}{4}x^2\right]_1^k$	A1		Both integrals fully correct Allow without limits	
			$\left(\frac{1}{2}k^2\ln k-\frac{1}{4}k^2\right)-\left(\frac{1}{2}\ln 1-\frac{1}{4}\right)=\frac{1}{2}k^2\ln k-\frac{1}{4}k^2+\frac{1}{4}$ Area $=\left(k^2\ln k-k^2+k\right)-\left(\frac{1}{2}k^2\ln k-\frac{1}{4}k^2+\frac{1}{4}\right)$				
			$=\frac{1}{2}k^2\ln k-\frac{3}{4}k^2+k-\frac{1}{4}$	A1		Cao	

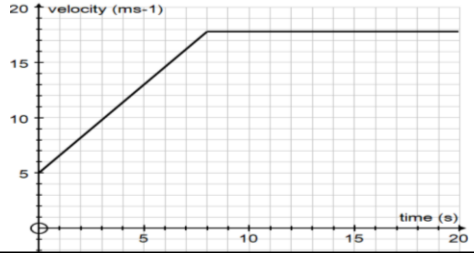
Question			Answer	Marks	AOs		Guidance
11	(i)		Component of weight down the plane $4.7g\sin 60°$ Equilibrium equation $T = 4.7g\sin 60°$ $= 39.889...$ so $T = 39.9$ to 3 sf	B1 E1 [2]	2.1 3.3	**AG** Award if seen Must be clear that 39.9 N is the tension and not just component of weight	
	(ii)		Resolve perpendicular to the slope N is the normal reaction between plane and block B $N = 4g\cos 25°$ Resolve up the slope $T - F - 4g\sin 25° = 0$ On the point of sliding so $F = \mu N = \mu \times 4g\cos 25°$ $\mu = \frac{4.7g\sin 60° - 4g\sin 25°}{4g\cos 25°} = 0.656$ to 3sf	B1 M1 A1 M1 A1 [5]	1.1a 3.3 1.1b 3.1b 1.1b	Need not be evaluated here $[\approx 35.5]$ Allow only sign errors F need not be evaluated here $[\approx 23.3]$ Do not allow for $F \le \mu N$ unless = used subsequently. FT their values. FT (notice this answer is 0.657 if 39.9 used for T)	If only values are seen used, it must be clear that the values used are friction and normal reaction.

Question			Answer	Marks	AOs		Guidance
12	(i)		C is (1, -1)	**B1** **[1]**	**1.1a**	Cao	
	(ii)	A	EITHER Substitute $y = \frac{3}{4}x - 8$ into the equation of the circle $(x-1)^2 + \left(\frac{3}{4}x - 8 + 1\right)^2 = 25$ $x^2 - 8x + 16 = 0$ EITHER $(x-4)^2 = 0$ OR Discriminant = $(-8)^2 - 4 \times 1 \times 16 = 0$ So the equation has a repeated root so the line is a tangent	**M1** **M1** **A1** **A1** **[4]**	**3.1a** **1.1a** **1.1b** **2.2a**	**AG** Attempt to eliminate one variable Attempt to expand and collect terms to obtain 3 term quadratic expression A correct 3 term quadratic Clearly argued	
	(ii)	A	OR Substitute $x = \frac{4}{3}y - \frac{32}{3}$ into the equation of the circle $\left(\frac{4}{3}y - \frac{32}{3} - 1\right)^2 + (y+1)^2 = 25$ $y^2 + 10y + 25 = 0$ EITHER $(y+5)^2 = 0$ OR Discriminant = $10^2 - 4 \times 1 \times 25 = 0$ So the equation has a repeated root so the line is a tangent	**M1** **M1** **A1** **A1** **[4]**	**3.1a** **1.1a** **1.1b** **2.2a**	**AG** Attempt to eliminate one variable Attempt to expand and collect terms to obtain 3 term quadratic expression A correct 3 term quadratic Clearly argued	
		B	$x = 4$ and $y = -5$ so B is (4, -5)	**B1** **[1]**	**1.1a**	Cao	

Question			Answer	Marks	AOs		Guidance
	(iii)		$\angle CAD = \angle CBD = 90°$ (radius is perpendicular to the tangent) Gradient of AC $= \frac{2-(-1)}{5-1} = \frac{3}{4}$ Gradient of BC $= \frac{(-1)-(-5)}{1-4} = -\frac{4}{3}$ So AC is perpendicular to BC so $\angle ACB = 90°$ So ADBC is a rectangle Either AC = BC radius [=5] Or AD = BD equal tangents so ADBC is a square.	**B1** **B1** **B1** **[3]**	**2.1** **3.1a** **2.1**	Allow for one or other of these angles Complete proof **AG**	Allow up to B1, B1 for any two of these three pieces of evidence. Allow the final B1 only when the proof is complete and clearly argued.
			$\angle CAD = \angle CBD = 90°$ (radius is perpendicular to the tangent) Gradient of AC $= \frac{2-(-1)}{5-1} = \frac{3}{4}$ Gradient of BD is $\frac{3}{4}$ So AC is parallel to BD So ADBC is a rectangle AC = BC = radius so ADBC is a square.	**B1** **B1** **B1**		Allow for one or other of these angles Complete proof **AG**	
			$\angle CAD = 90°$ (radius is perpendicular to the tangent) AC = BC radius [=5] Gradient of AC $= \frac{2-(-1)}{5-1} = \frac{3}{4}$ Equation of AD is $y-2 = -\frac{4}{3}(x-5)$ So coordinates of D are (8, -2) Hence BD = 5 and AD = 5 So ABCD is a rhombus	**B1** **B1** **B1**		Gradient of AD must be found from the coordintes of A and C Complete proof	

Question			Answer	Marks	AOs		Guidance
	(iv)		E is the point (1, -6)	B1	2.1	May be implied	
			EITHER 	M1	3.1a	Right-angled triangle formed and use of arctan	
			$\theta = \arctan\left(\frac{3}{4}\right) = 0.6435$	A1	3.1a	oe	
			OR $BE = \sqrt{(4-1)^2 + (-5-(-6))^2} = \sqrt{10}$ Cosine rule in triangle BCE $\cos BCE = \frac{5^2+5^2-10}{2\times5\times5}\left[=\frac{40}{50}\right]$	(M1)		Using distance BC and the cosine rule	
			$\angle BCE = 0.6435...$	(A1)		oe	
			OR M is the midpoint of BE M is (2.5, -5.5) $BM = \sqrt{(4-2.5)^2 + (-5-(-5.5))^2} = \frac{1}{2}\sqrt{10}$ $\angle BCM = \arcsin\left(\frac{\frac{1}{2}\sqrt{10}}{5}\right) = 0.32175...$	(M1)		Using trig in triangle BCM or ECM Allow for $\angle BCM$	
			$\angle BCE = 0.6435...$	(A1)		Oe. Must be $\angle BCE$	
			Area sector = $\frac{1}{2}r^2\theta = \frac{1}{2}\times25\times2\times0.32175...$	M1dep	1.1a	Using the sector area formula	
			$= 8.04376$	A1 [5]	1.1b	FT their $\angle BCM$	

Question			Answer	Marks	AOs		Guidance
13	(i)		$f'(x)=\frac{1}{3}\left(27-8x^3\right)^{-2/3}\times(-24x^2)$ $\left[=\frac{-8x^2}{\left(27-8x^3\right)^{2/3}}\right]$ $f'(1.5)=-\frac{8\times1.5^2}{0}$ and dividing by zero zgives the error.	M1 A1 E1 [3]	1.1a 1.1 2.4	Using the chain rule Allow unsimplified Sufficient to say "can't divide by zero" oe	
	(ii)		$\left(27-8x^3\right)^{1/3}=27^{1/3}\left(1-\frac{8}{27}x^3\right)^{1/3}$ $=3\left(1+\left(\frac{1}{3}\right)\left(-\frac{8x^3}{27}\right)+\frac{\left(\frac{1}{3}\right)\left(-\frac{2}{3}\right)}{2!}\left(-\frac{8x^3}{27}\right)^2+\ldots\right)$ $=3-\frac{8x^3}{27}-\frac{64x^6}{2187}+\ldots$	B1 M1 A1 [3]	3.1a 1.1a 1.1b	Dealing with the 27 correctly Using the Binomial expansion substantially correctly Cao	
	(iii)		The binomial expansion is valid for $\left\lvert -8\frac{x^3}{27}\right\rvert<1$ $\lvert x\rvert<1.5$ and the limits of the integral are completely in this interval.	B1 E1 [2]	2.4 2.3	Allow unsimplified but must use correct modulus notation or equivalent Must indicate that the limits of the integral lie in their interval for which the expansion is valid.	
	(iv)		$\frac{0.25}{2}\left(3+2.6684+2\left(2.9954+2.9625+2.8694\right)\right)$ $=\frac{0.25}{2}\times23.3224=2.9153$	B1 M1 A1 [3]	1.1a 1.1b 1.1b	$h=0.25$ used For sum in the bracket – condone one slip. Allow for 2.92 or better	Values from candidates own calculators may differ in the last decimal place.
	(v)		There is area between the curve and the top of the trapezia, so some area is missing from the estimate.	E1 [1]	2.4	Allow for any sensible explanation eg the trapezia are under the curve	"The curve is concave downwards" on its own is not quite enough

Question			Answer	Marks	AOs		Guidance
14	(i)		$u=5,\ v=11.4,\ t=4$ $a=\frac{v-u}{t}=\frac{11.4-5}{4}=1.6$ $v=5+1.6t$	M1 A1 A1 [3]	3.1b 1.1b 3.3	Using *suvat* equation(s) leading to value for *a* Any form FT their *a*	
	(ii)		The car would not be able to accelerate indefinitely – the velocity would become too large	E1 [1]	3.5b		
	(iii)		When $v=17.8$ $t=\frac{17.8-5}{1.6}=8$ 	B1 G1 G1 [3]	1.1a 1.1a 3.5c	Calculation or point on graph labelled at t = 8 Two line segments with one horizontal Axes labelled. (0, 5) and constant speed 17.8 clear on vertical scale	Mark intent for 17.8 – allow for a linear scale beyond 17.8
	(iv)		Dividing area into sections Area under trapezium = $\frac{1}{2}(5+17.8)\times 8=91.2$ Area rectangle $12\times 17.8=213.6$ Total displacement = 304.8 m	M1 A1 A1 [3]	3.1b 1.1a 1.1b	 May be found as sum of areas. May be implied by correct total FT their distance found for first 8s	FT their graph if linear for M1 A0 for a triangle or trapezium area 213.6 must be added to another distance
	(v)		When $t=4\ \ v=5+0.3\times 4^2-0.05\times 4^3=11.4\ \mathrm{m\,s^{-1}}$ Which matches the given value	B1 [1]	3.4	Allow without comment	
	(vi)		$\frac{dv}{dt}=0.6\times 2t-0.05\times 3t^2\ \left[=1.2t-0.15t^2\right]$ When $t=8\ \ v=1.2\times 8-0.15\times 64=0$ Acceleration is zero at $t=8$ which means that the car reaches its maximum speed without the sudden change in acceleration in model A.	M1 A1 E1 [3]	1.1a 3.2a 3.2a	Need not be simplified Must mention acceleration Must compare with model A	Final mark can be awarded independently for a statement about change in acceleration as long as supported by some numerical evidence

Question			Answer	Marks	AOs		Guidance
	(vii)		EITHER $\int_0^8 \left(5+0.6t^2-0.05t^3\right)\,\mathrm{d}t=\left[5t+0.2t^3-0.0125t^4\right]_0^8$ =91.2 m which is same as model A for the first 8 s Distance is the same for the remainder of the time So this is the same as model A at t = 20	**M1** **A1** **E1** **[3]**	**2.1** **1.1b** **2.1**	 **BC** Must consider to t = 20	Allow for correct definite integral stated and calculator used. Also allow M1A1 for $5\times8-0.2\times8^3-0.0125\times8^4$ seen
			OR $\int_0^8 \left(5+0.6t^2-0.05t^3\right)\,\mathrm{d}t=\left[5t+0.2t^3-0.0125t^4\right]_0^8$ =91.2 m Distance at 17.8 ms^{-1} 213.6 Total distance 304.8m [which is the same as model A]	 **M1** **A1** **A1**		 **BC** Must consider to t = 20	Allow for correct definite integral stated and calculator used.

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A LEVEL

Examiners' report

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/01 Summer 2018 series

Version 1

Contents

Introduction

Our examiners' reports are produced to offer constructive feedback on candidates' performance in the examinations. They provide useful guidance for future candidates. The reports will include a general commentary on candidates' performance, identify technical aspects examined in the questions and highlight good performance and where performance could be improved. The reports will also explain aspects which caused difficulty and why the difficulties arose, whether through a lack of knowledge, poor examination technique, or any other identifiable and explainable reason.

Where overall performance on a question/question part was considered good, with no particular areas to highlight, these questions have not been included in the report. A full copy of the question paper can be downloaded from OCR.

Paper H640/01 series overview

This was the first paper for this new A Level and all the candidates had prepared for this examination in one year. The marks were generally very good as many candidates are also Further Mathematics candidates. This paper contributes 36.4% of the total A Level and assesses content from Pure Mathematics and Mechanics.

Candidates showed a good understanding on Mechanics as well as Pure Mathematics. Candidates were well able to cope with extended answers (for example question 10). Questions requiring proof or an explanation were less well answered (for example question 12(iii) 13(v) and 14(ii)).

To do well in this component, candidates need to be able to apply their knowledge of the syllabus content in a variety of modelling contexts and to make efficient use of calculator technology.

Section A overview

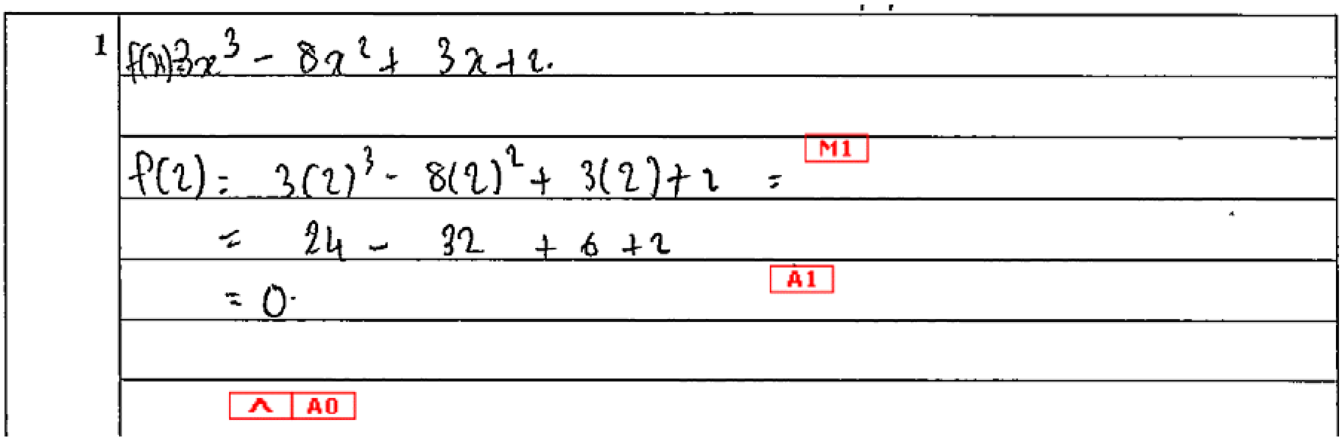
Section A questions are designed to give all candidates an opportunity to do some of the questions on the paper as they require little reading or interpretation. Most candidates did very well in Section A.

Question 1

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. **[3]**

There were many good answers and candidates could choose whether to use the factor theorem or to divide. Some candidates lost the mark as they presented the evidence but did not write that $f(2)=0$ implied that $(x-2)$ is a factor, or that no remainder implied that $(x-2)$ is a factor.

Exemplar 1



1 $f(x) 3x^3 - 8x^2 + 3x + 2.$

$f(2) = 3(2)^3 - 8(2)^2 + 3(2) + 2 =$ M1

$= 24 - 32 + 6 + 2$

$= 0.$ A1

^ A0

This exemplar shows why some candidates were only credited 2 of 3 marks.

Question 2

2 By considering a change of sign, show that the equation $e^x - 5x^3 = 0$ has a root between 0 and 1. **[2]**

This question was generally well answered with only a few candidates making an arithmetical error that cost a mark.

Question 3

3 **In this question you must show detailed reasoning.**

Solve the equation $\sec^2\theta + 2\tan\theta = 4$ for $0° \leqslant \theta < 360°$. **[4]**

Candidates who used the identity $\sec^2\theta = 1 + \tan^2\theta$ generally went on to obtain most of the marks. Only a few candidates tried to rewrite the equation in terms of $\cos\theta$ as this is a much more difficult method requiring both sides to be squared and spurious solutions eliminated. Candidates did not get far enough into this method to obtain the method mark.

Question 4 (i)

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\mathrm{m\,s^{-1}}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. **[2]**

Most candidates successfully used the *suvat* equations to reach the correct answer.

Question 4 (ii)

(ii) Find the magnitude of the force that Rory applies. **[2]**

Newton's second law was well understood and most candidates were successful here.

Question 5 (i)

5 The position vector $\mathbf{r}$ metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time t. **[2]**

Most candidates used vector notation accurately and successfully differentiated to obtain a correct expression for velocity.

Question 5 (ii)

(ii) Determine whether the particle is ever stationary. **[2]**

This was typically well answered with most candidates realising the requirement for both components to be zero at the same time.

Misconception It is not sufficient to equate the components and solve to find $t = 3$. From this starting point, candidates would need to check that the components were zero to achieve the method mark.

Question 6 (i)

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. **[2]**

Some candidates used a brute force method, writing out the complete list of monthly payments and adding them. Most successfully identified this as an arithmetic series and used the correct formula to find the sum of 24 terms.

Partial credit was not awarded where a candidate found the 24th term but did not then attempt to find the sum total.

Question 6 (ii)

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

Many candidates realised that the 12% increase could be achieved by multiplying by 1.12 which leads to a geometric series. The value 1.12 had to be seen in part (ii) for the mark to be credited.

AfL Even if 1.12 was used in part (iii) this mark could not be credited if 1.12 was not seen in part (ii).

Exemplar 2

6(ii)	Because the amount saved is going up in a common ratio, and the 50 it is not the same difference each time. ^ BO

Some candidates were not credited the mark as their answer was too vague, as in the exemplar above.

Question 6 (iii)

(iii) Determine whether Baraka saves more in two years than Aleela. **[3]**

Again the formula for the total of terms was needed to earn the method mark. The final mark was credited to candidates who compared their totals and follow-through was available to those who had made an arithmetic mistake but not to those who had not earned the method marks in both part (i) and part (iii).

Section B overview

Section B contains longer questions and more problem solving than Section A. The questions are graded in difficulty and this was reflected in the marks credited, although some candidates who were well prepared in Mechanics found question 14 straightforward.

Question 7 (i)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

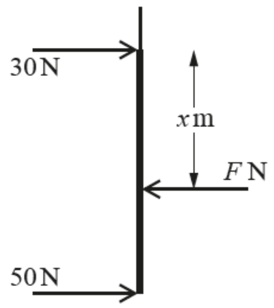


Fig. 7

(i) Find the value of F. **[1]**

This was a routine mark for which almost all candidates were credited, the only mistake seen was to find the difference between 50 N and 30 N,

Question 7 (ii)

(ii) Find the value of x. **[2]**

Most candidates successfully took moments about the top of the rod to easily obtain the correct answer. Candidates that attempted to take moments about the centre of the rod often encountered problems with their positioning of F with respect to the centre.

Question 8 (i)

8 **(i)** Show that $8\sin^2 x\cos^2 x$ can be written as $1-\cos 4x$. **[3]**

This question posed considerable difficulty to many candidates. Some had success by writing both $\sin^2 x$ and $\cos^2 x$ in terms of $\cos 2x$ and multiplying out. Many then did not go on to complete the proof. The first 2 marks were credited for any two applications of the double angle formulae and the final mark only when there was a convincing proof so many candidates were credited 2 out of 3 marks.

Question 8 (ii)

(ii) Hence find $\int \sin^2 x\cos^2 x\,dx$. **[3]**

Most candidates realised they needed to use the identity from part (i) although many omitted the factor of $\frac{1}{8}$. Some candidates lost the final mark for omitting the $+c$.

Question 9 (i)

9 A pebble is thrown horizontally at $14\,\mathrm{m\,s^{-1}}$ from a window which is $5\,\mathrm{m}$ above horizontal ground. The pebble goes over a fence $2\,\mathrm{m}$ high $d\,\mathrm{m}$ away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

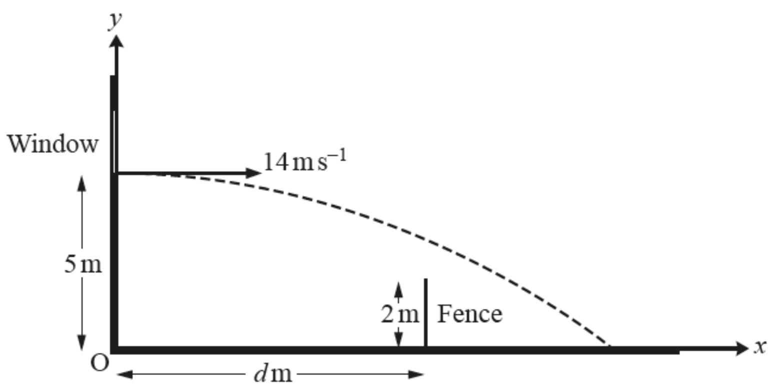


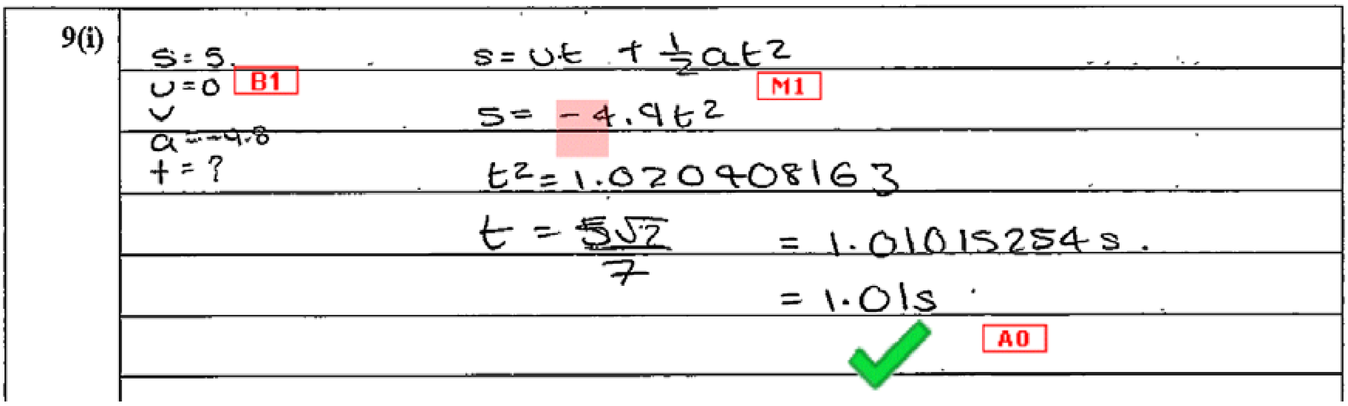
Fig. 9

(i) Find the time the pebble takes to reach the ground. **[3]**

Most candidates realised that the initial velocity in the vertical direction was zero and successfully completed this question.

AfL It is important to make use of a consistent sign convention, with acceleration and displacement either both positive or both negative. A clear statement of the positive direction at the start of the answer helps avoid problems.

Exemplar 3



9(i)

$s = 5$ B1

$u = 0$

v

$a = -9.8$

$t = ?$

$s = ut + \frac{1}{2}at^2$ M1

$s = -4.9t^2$

$t^2 = 1.020408163$

$t = \frac{5\sqrt{2}}{7} = 1.01015254\text{ s}$.

$= 1.01\text{s}$

A0

This candidate was credited B1 using u = 0 and M1 for the equation with a sign error. Although the correct answer is seen, it comes from incorrect working and was therefore not credited the final mark. This candidate obviously recognised that there was an issue with the working and should have gone back to identify and correct their mistake ($s = -5$ or $a = +9.8$).

Question 9 (ii)

(ii) Find the cartesian equation of the trajectory of the pebble. **[4]**

Many correct answers were seen. The most common error was to omit the initial height of the pebble. The origin is given in the question, so the correct equation is $y = 5 - 4.9t^2$.

AfL Take careful note of the origin and remember to include the initial position in the equations.

Question 9 (iii)

(iii) Find the range of possible values for d. **[3]**

The question was designed so that the simplest way to answer this was to substitute $y = 2$ in the equation of the trajectory leading to $x = 10.95$. Common sense was enough to use this as a boundary value for the inequality – the pebble would go over the wall if it were nearer the window than that value.

Many candidates went back to the original model, found the time to drop to the height of the wall, used that to work out the boundary value for d, and received full credit.

Question 10

10 Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

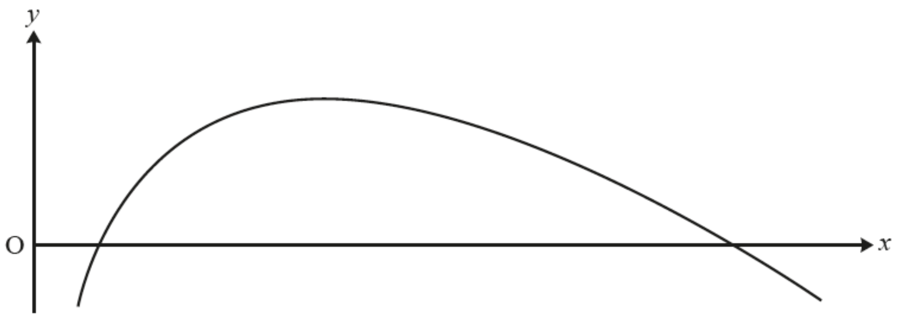


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

This is an exemplar of a question requiring an extended answer – there are 4 method marks in the scheme. Candidates had to structure their answer. Had a value for *k* been given, the definite integral would have been possible on many calculators and the question may have become a "detailed reasoning" question.

Successful candidates generally used integration by parts with $\frac{dv}{dx} = k - x$. It needed much more work to expand the bracket and split the integral in two. Some candidates lost the final mark, as they did not tidy up their answer so had too many terms.

AfL

In an unstructured question like this one, do not give up or leave it blank because you do not know how to calculate the limits. Find the indefinite integral to make sure of 4 out of 8 marks. Using incorrect limits could also have been credited a method mark.

Question 11 (i)

11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

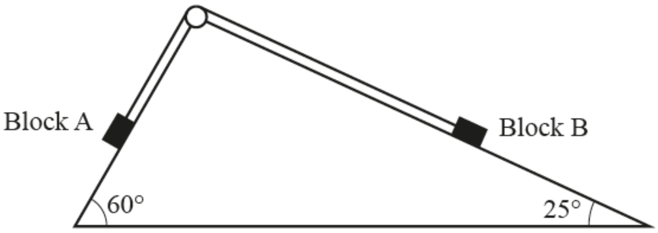


Fig. 11

(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

This was generally well answered but some candidates lost a mark as all they had written was a component of weight and it was not clear that they had equated that to the tension. Since the answer was given in the question, the response needed to be a full mathematical justification to show the given answer.

AfL Make sure that you set up an equilibrium equation even if there are only two terms.

Question 11 (ii)

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

The problem solving element of this question stems from the lack of help that candidates were given in structuring their answer. They had to realise that they had to calculate the normal reaction and the frictional force before they could calculate the coefficient of friction. Some answers were very fragmented with very little help given by candidates to the examiner who were not always able to tell whether finding the normal reaction and the frictional forces had been attempted.

Question 12 (i)

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

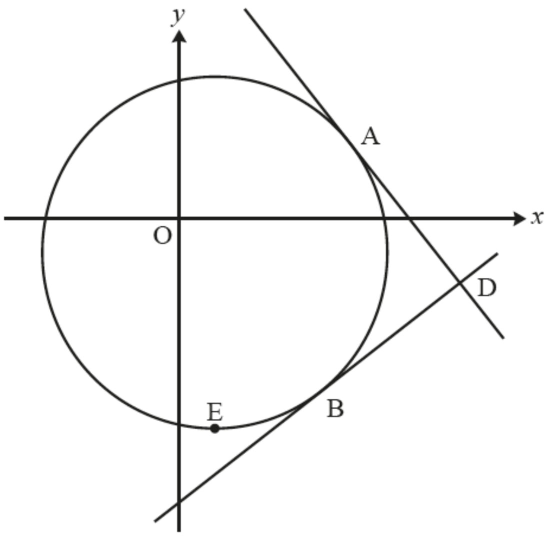


Fig. 12

(i) Write down the coordinates of C, the centre of the circle. **[1]**

Most candidates were correct here.

Question 12 (ii) (A)

(ii) (*A*) Show that the line $4y=3x-32$ is a tangent to the circle. **[4]**

There was quite a lot of confusion in this question between the tangent to the circle at A (whose equation is not given nor required) and the line $4y=3x-32$ which can be proved to be the tangent to the circle at B. Some candidates set out to find the equation of the tangent at A hoping to obtain the given equation.

Successful candidates solved the equation of the line and the circle simultaneously although many struggled with the fractions that resulted.

Question 12 (ii) (B)

(*B*) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. **[1]**

There was only one mark here as the *x*-coordinate had normally been found in part A, so there was only the *y*-coordinate to find.

Question 12 (iii)

(iii) Prove that ADBC is a square. **[3]**

This proved quite challenging for many candidates. The first 2 marks were given for two facts that form part of a complete proof, and the third only when a complete proof was seen. Some candidates assumed that AD and BD were perpendicular in order to find the equation of AD and subsequently used their equation to complete their answer – but this is not a proper proof. Candidates needed to show they had used the gradient of AC or the derivative of the circle to find the gradient of the tangent at A.

Question 12 (iv)

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

The problem solving aspect of this question involved the need to find the angle ECB before the formula for the area of the sector could be used. Many candidates managed to write down the coordinates of E for the first mark. There were several valid methods of finding the required angle but it was often difficult to follow candidates thinking unless the correct answer was obtained. The second method mark was dependent on the first so that it was not enough just to guess an angle and use that.

AfL Write a few words of explanation to the examiner, or clear headings, indicating your method so that you can be given a method mark even if your answer is not quite right. Two method marks and the follow-through answer mark may depend on it.

Question 13 (i)

13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27 - 8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. **[3]**

This was well answered as most candidates used the chain rule successfully and realised that substituting x = 1.5 gives a zero in the denominator

Question 13 (ii)

(ii) Find the first three terms of the binomial expansion of $f(x)$. **[3]**

Many candidates dealt successfully with the 27, but when that was done without clear working shown, could cost 2 marks here. Some candidates simplified their coefficients early and incorrectly, so it was not always clear that the binomial expansion had been used, costing the method mark.

AfL Make your method clear by writing down the factors of each term before simplifying.

Question 13 (iii)

(iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

One of the assessment objectives in the specification is to test the ability of a candidate to assess the validity of an argument as in this question. Not many candidates realised that the key to this explanation was to find the range of values for which the binomial expansion is valid. The limits lie well within the valid range so the method is valid.

Question 13 (iv)

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. **[3]**

This was generally done very well.

Question 13 (v)

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27-8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 13.

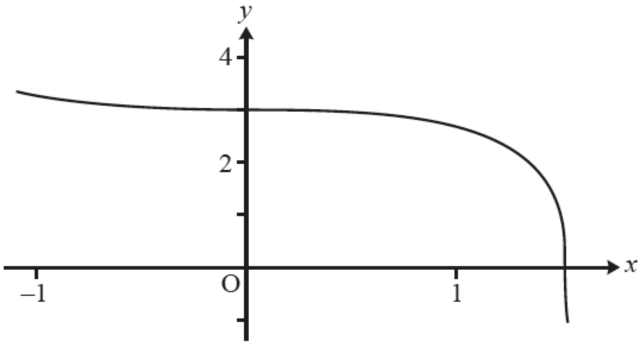


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

Most candidates were able to explain this clearly. Some had learned that a curve being concave downwards would give an underestimate but gave no indication as to why that would be, so lost the mark.

AfL Clear annotated sketches can support a mathematical explanation better than an extended written response.

Question 14 (i)

14 The velocity of a car, $v\,\mathrm{m\,s^{-1}}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\mathrm{m\,s^{-1}}$ and it accelerates to $11.4\,\mathrm{m\,s^{-1}}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

The key to this question was to calculate the acceleration of the car. The required expression is then found by substituting the values for *u* and *a* into the equation $v = u + at$. Many fully correct answers were seen.

Question 14 (ii)

(ii) Explain why this model is not appropriate in the long term. **[1]**

Candidates were required to recognise the limitations of this model. Most successful answers stated that the velocity would eventually get much too big or that the car would have to slow down or stop at some point.

Question 14 (iii)

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\text{m}\,\text{s}^{-1}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

Most candidates correctly had a graph consisting of two line segments but a common error was to begin the graph at the origin when the initial velocity was $5\,\text{ms}^{-1}$. Some did not fully label their graph so lost a mark.

AfL Make sure the axes are labelled and that all key points are clearly indicated on the graph.

Question 14 (iv)

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

Many candidates were successful in finding the area under their graph with only a few arithmetical errors.

Question 14 (v)

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

This mark was credited for seeing the substitution of $t = 4$ into the equation. It would have been good to see this followed by a comment that the value was close to the given value.

Question 14 (vi)

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

Some candidates were very vague in their answer here and were not credited many marks. There is a very clear instruction that it is the value of acceleration that is needed, so 2 of the 3 marks were given for finding this. Many candidates were able to comment that model B gives a gradual change in acceleration as it approaches the maximum speed avoiding the very sudden change seen in model A.

Question 14 (vii)

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

Many candidates realised that the distance was the definite integral that gave the distance travelled in the first 8s. It would have been sufficient to clearly write the integral with its limits and use a calculator to evaluate it. Most candidates decided to give a full solution with the substitution of limits made clear. Only a few candidates omitted the part of the journey beyond 8s.

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Exemplar Candidate Work

MATHEMATICS B (MEI)

H640
For first teaching in 2017

H640/01 Summer 2018 examination series

Version 1

Contents

Introduction

These exemplar answers have been chosen from the summer 2018 examination series.

OCR is open to a wide variety of approaches and all answers are considered on their merits. These exemplars, therefore, should not be seen as the only way to answer questions but do illustrate how the mark scheme has been applied.

Please always refer to the specification http://www.ocr.org.uk/Images/308740-specification-accredited-a-level-gce-mathematics-b-mei-h640.pdf for full details of the assessment for this qualification. These exemplar answers should also be read in conjunction with the sample assessment materials and the June 2018 Examiners' report or Report to Centres available from Interchange https://interchange.ocr.org.uk/Home.mvc/Index

The question paper, mark scheme and any resource booklet(s) will be available on the OCR website from summer 2019. Until then, they are available on OCR Interchange (school exams officers will have a login for this and are able to set up teachers with specific logins – see the following link for further information http://www.ocr.org.uk/administration/support-and-tools/interchange/managing-user-accounts/).

It is important to note that approaches to question setting and marking will remain consistent. At the same time OCR reviews all its qualifications annually and may make small adjustments to improve the performance of its assessments. We will let you know of any substantive changes.

Question 1

1 Show that $(x-2)$ is a factor of $3x^3-8x^2+3x+2$. **[3]**

Exemplar 1 — 3 marks

1

$$3x^2 - 2x - 1$$
$$x-2 \,\big)\, 3x^3 - 8x^2 + 3x + 2$$
$$-\; 3x^3 - 6x^2$$
$$-2x^2 + 3x$$
$$-\; -2x^2 + 4x$$
$$-x + 2$$
$$-\; -x + 2$$
$$0$$

$3x^3 - 8x^2 + 3x + 2 = (x-2)(3x^2-2x-1)$

If $x-2$ is a factor, sub $x=2$ into $3x^3-8x^2+3x+2=0$

(factor theorem)

$3(2)^3 - 8(2)^2 + 3x + 2 = 24 - 32 + 6 + 2 = 0$ $\therefore$

~~so~~ $x-2$ is a factor.

Examiner commentary

There are two valid methods of answering this question and this candidate uses both of them perfectly. Generally, this is not a good idea and where two attempts are made at the same question, the examiner will usually take the second answer as the one the candidate intended to be marked. Notice the examiner could have specified which method was to be used in a question like this and using the other method would be likely to earn no marks.

This candidate also has a very clear conclusion following their evidence – some candidates lost a mark here as they left out their conclusion (see exemplar provided in the examiner's report).

Question 2

2 By considering a change of sign, show that the equation $e^x - 5x^3 = 0$ has a root between 0 and 1. **[2]**

Exemplar 1 — 1 mark

2

Let $f(x) = e^x - 5x^3$

$f(0) = e^0 = 1$ M1 $+ve$

$f(1) = e^1 - 5 = -4$ A0 $-ve$

There is a change in sign of f(x) between 0 and 1, indicating that f(x), a continuous function, passes through the x-axis between 0 and 1 ∴ there is a root between 0 and 1 for f(x).

Examiner commentary

This was quite a straightforward question and many candidates got full marks for this. This candidate has made a mistake in the arithmetic so loses the A mark. Notice that this is a very good example of the argument that was also needed to obtain the A mark.

Question 3

3 In this question you must show detailed reasoning.

Solve the equation $\sec^2\theta + 2\tan\theta = 4$ for $0° \leqslant \theta < 360°$. **[4]**

Exemplar 1

3 marks

3

$\sec^2\theta + 2\tan\theta = 4$ since: $\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 + 2\tan\theta = 4$ M1 then: $\tan^2\theta + 1 = \sec^2\theta$

$\tan^2\theta + 2\tan\theta + 1 = 4$

$\tan^2\theta + 2\tan\theta - 3 = 0$

$(\tan\theta - 1)(\tan\theta + 3) = 0$ M1

$\tan\theta = 1$ $\tan^{-1}(1) = 45°$ $45 + 180 = 225°$

$\tan\theta = -3$ $\tan^{-1}(-3) = -71.56$ $-71.56 + 180 = 108.43°$

$= 45°, 108.4°, 225°$ A1

^

Examiner commentary

This question required the use of a trigonometrical identity leading to a quadratic equation in tan θ. Each value leads to two roots in the required interval. This candidate has a very clear method for finding the values for tan θ and finds two values of θ for each. One principal value is outside the range so the candidate does not realise that there is another value to be found. This is an example of a place where smart use of a calculator to check the answers might have picked up the missing value. Notice the question required "**detailed reasoning**", so no marks would be obtained for finding the values only using a calculator with no method seen.

Question 4 (i) and (ii)

4 Rory pushes a box of mass 2.8 kg across a rough horizontal floor against a resistance of 19 N. Rory applies a constant horizontal force. The box accelerates from rest to $1.2\,\mathrm{m\,s^{-1}}$ as it travels 1.8 m.

(i) Calculate the acceleration of the box. **[2]**

(ii) Find the magnitude of the force that Rory applies. **[2]**

Exemplar 1

(i) 2 marks

4(i)

$m = 2.8\,\mathrm{kg}$

s 1.8
u 0
v 1.2
a
T ×

$v^2 = u^2 + 2as$

$a = \dfrac{v^2 - u^2}{2s}$

$= \dfrac{1.2^2}{2 \times 1.8}$

$a = 0.4\ \mathrm{ms^{-2}}$

(ii) 2 marks

4(ii)

$F = ma$ F

$F_o - 19 = 2.8 \times 0.4$

$F = \cancel{20}.\ 1.12 + 19$

$= 20.12\ \mathrm{N}$

Examiner commentary

This is a good example of best practice in Mechanics. In part (i) the candidate has provided a clear diagram showing the forces on the object and a list of the variables used in the *suvat* equations with a cross against the variable, which is neither given nor required. This helps to choose the best equation to use to answer the question efficiently. The equation is clearly stated and the substitution of the values clearly seen. In part (ii) Newton's second law is clearly stated in the form $F = ma$, but in a question like this one where F is one of the forces and not the resultant force, it can sometimes cause confusion to candidates. Often it is better to state Newton's second law as "resultant force = *ma*". This candidate understands this as can be seen from their correct second line of working and correct answer.

Question 5 (ii)

5 The position vector **r** metres of a particle at time t seconds is given by

$$\mathbf{r} = (1 + 12t - 2t^2)\mathbf{i} + (t^2 - 6t)\mathbf{j}.$$

(i) Find an expression for the velocity of the particle at time t. **[2]**

(ii) Determine whether the particle is ever stationary. **[2]**

Exemplar 1

(ii) 1 mark

5(ii)	
	when it is stationary $v = 0$ ~~[illegible]~~
	i j components equal zero
	i components $\cancel{0 = 12 - 4t}$ M1
	$\cancel{-12 = -4t}$ $2t - 6 = 0$
	$\cancel{3 = t}$ $2t = 6$
	$t = 3$
	at $t=3$ the particle is stationary ^ A0

Examiner commentary

This candidate works with one component only – they may have realised that both components were zero but there is no evidence of that for the examiner to see. Having found t=3 using the j component, the candidate should have verified that when t=3 the i component (12-4t)=0.

Exemplar 2

(ii) 2 marks

5(ii)

~~$a_k = 50 \times 1.12^{k-1}$ is a geometric sequence with first term 50 and common ratio 1.12~~

~~$a_{24} = 50 \times 1.12^{23} = £677.62$ in 2 years, which is more than £510 ∴ Bartka saves more than Aleela in 2 years. (£167.62 more).~~

Particle is stationary when $\underline{v} = 0\underline{i} + 0\underline{j}$

$12 - 4t = 0$ —— 1

$2t - 6 = 0$ —— 2

$12 - 4t = 2t - 6$

$6t = 18$

$t = 3$

Particle is stationary when $t = 3$.

Examiner commentary

In this exemplar, the candidate has made clear that both components must equal zero, and that the components must be equal at t=3. Ideally a full solution should have the substitution t=3 back into each component to verify that the object is stationary and not travelling in the direction of vector $\binom{1}{1}$, but the benefit of the doubt can be given in this simple linear situation.

Question 6 (i), (ii) and (iii)

6 Aleela and Baraka are saving to buy a car. Aleela saves £50 in the first month. She increases the amount she saves by £20 each month.

(i) Calculate how much she saves in two years. **[2]**

Baraka also saves £50 in the first month. The amount he saves each month is 12% more than the amount he saved in the previous month.

(ii) Explain why the amounts Baraka saves each month form a geometric sequence. **[1]**

(iii) Determine whether Baraka saves more in two years than Aleela. **[3]**

Exemplar 1

(i) 0 marks

6(i)	
	$50 \quad 70 \quad 90$ $\qquad a+(n-1)d$
	$+20$ $\qquad 50+(n-1)20$
	two years = 24 months $\qquad 50+(24-1)\times 20 = 510$
	$(23\times 20)+50 = £510$

(ii) 1 mark

6(ii)	
	$50 = a$ $\qquad ar^{n-1}$
	$1.12 = r$
	it forms a geometric sequence as the multiplier
	can be 1.12 to show a 12% increase

(iii) 0 marks

6(iii)	$50 \times 1.12^{n-1}$
	$50 \times 1.12^{24-1}$
	$50 \times 1.12^{23} = £677.617$
	∴ Baraka saves more money in two years than Aleela

Examiner commentary

This exemplar shows that the candidate has correctly identified the situation as one giving an arithmetic series but has evaluated the 24th term of the sequence and not the sum of the 24th term. Generally, candidates who lost the marks here also lost the marks in part (iii) by making the same mistake. The candidate answered part (ii) nicely; making clear the link between the 12% given in the context of the question and 1.12 needed for the geometric sequence formulae.

Exemplar 2

(i) 1 mark

6(i)	50 + 70 + 90 + 110 + 130 + 150
	+ 170 + 190 + 210 + 230 + 250 + 270
	+ 290 + 310 + 330 + 350 + 370
	+ 390 + 410 + 430 + 450 + 470 + 490
	+ 510
	M1
	= £510 A0

(ii) 0 marks

6(ii)

The reason the amount banka saves each month forms a geometric sequence is because it has a common ratio and it forms a consecutive pattern increasing through n^{th} term. ~~e.g.~~

~~$A + 2$~~

^ B0

(iii) 0 marks

6(iii)

Aleela saves £510 ^ M0

Aleela saves more

Examiner commentary

In part (i) the candidate has used a brute force method, writing out the complete list of monthly payments; earning the method mark. In part (ii), the question asked candidates to explain their reasoning and so a detailed answer was needed. This candidate knows the definition of a geometric series depended on having a common ratio but without the value 1.12 there was no link made between the 12% increase in the context and the common ratio in the formulae. This inability to use the formulae in context is reflected in the lack of response in part (iii).

Question 7 (i) and (ii)

7 A rod of length 2 m hangs vertically in equilibrium. Parallel horizontal forces of 30 N and 50 N are applied to the top and bottom and the rod is held in place by a horizontal force F N applied x m below the top of the rod as shown in Fig. 7.

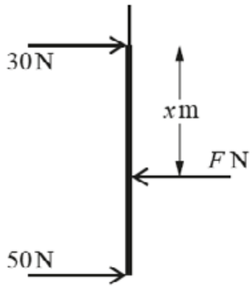


Fig. 7

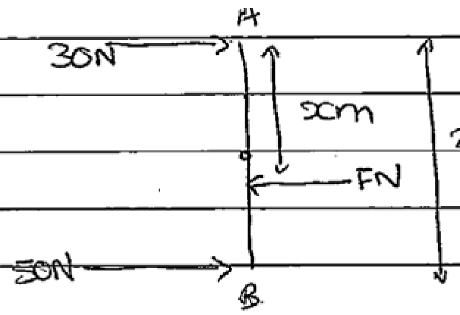
(i) Find the value of F. **[1]**

(ii) Find the value of x. **[2]**

Exemplar 1

(i) 0 marks

7(i)



moments about A =>

$50 \times 2 = F \times x$

$F = 20\text{N}$

(ii) 1 mark

7(ii)

$100 = 20x$ M1 FT

$x =$

Examiner commentary

This candidate generated the correct equation with two unknowns for the moments about A, unfortunately subtracting the given forces to find *F* rather than adding. The fact that they did not complete part (ii) suggests that they realised that *x*=5 m did not make sense. The exemplar shows the importance of taking care with signs.

Exemplar 2

(ii) 0 marks

7(ii)	
	$M(\text{centre}): 30 - 50 + F(1-x) = 0$ M0
	$F - Fx = 20$
	$\cancel{80}\ 80x = 20$
	$80x = 60$
	$x = 0.75\text{m}$

Examiner commentary

To find the distance, moments must be taken about any point. This candidate chose to take the middle of the rod so the distances for the 30 N and 50 N forces are 1 m. Numerically this is a correct equation but dimensionally it is not correct. The equation has two terms which are forces and the third term which is a moment, so this was credited 0/2 marks.

Question 8 (i) and (ii)

8 (i) Show that $8\sin^2 x\cos^2 x$ can be written as $1-\cos 4x$. [3]

(ii) Hence find $\int \sin^2 x\cos^2 x\,dx$. [3]

Exemplar 1

(i) 1 mark

8(i)

$8\sin^2 x\cos^2 x \Rightarrow$

double angle formula $\Rightarrow$ $\cos 2x = \cos^2 x - \sin^2 x$

$\sin^2 x = \cos^2 x - \cos 2x$

~~$\sin 2x = 2\cos x\sin x$~~

~~$2(\sin 2x)^2 = 4\cos^2 x\sin^2 x$~~ M1

~~$2(\sin 2x)^2 = 8\cos^2 x\sin^2 x$~~

8

(ii) 2 marks

8(ii)

$\int \frac{1-\cos 4x}{8}\,dx = \frac{1}{8}\int 1-\cos 4x\,dx$

$= \frac{1}{8}\left[x + \frac{1}{4}\sin 4x\right]$ M1 A1

$= \frac{1}{8}x + \frac{1}{32}\sin 4x.$ ^

Examiner commentary

This proof in part (i) is quite tricky – there are several correct ways of doing it shown in the mark scheme and other long-winded methods were also seen in scripts. This exemplar shows one step from a correct proof and is credited M1 even though the work is crossed out, as it has not been replaced with anything else. The candidate shows good exam technique in part (ii), using the given answer from (i) even though they had not been able to establish the result themselves. A mark is lost, however, as the $+c$ term is not included in the answer.

Exemplar 2

(ii) 2 marks

8(ii)	$\int \sin^2 x \cos^2 x \, dx$
	$= \int 1 - \cos 4x \, dx$ A0
	$= x - \frac{1}{4} \sin 4x + c$ M1 A1

Examiner commentary

This candidate also used the given answer from part (i) although they had made no attempt to prove it. This exemplar shows a common error – the factor of $\frac{1}{8}$ is missing, losing one of the accuracy marks.

Question 9 (i), (ii) and (iii)

9 A pebble is thrown horizontally at $14\,\text{m s}^{-1}$ from a window which is 5 m above horizontal ground. The pebble goes over a fence 2 m high d m away from the window as shown in Fig. 9. The origin is on the ground directly below the window with the x-axis horizontal in the direction in which the pebble is thrown and the y-axis vertically upwards.

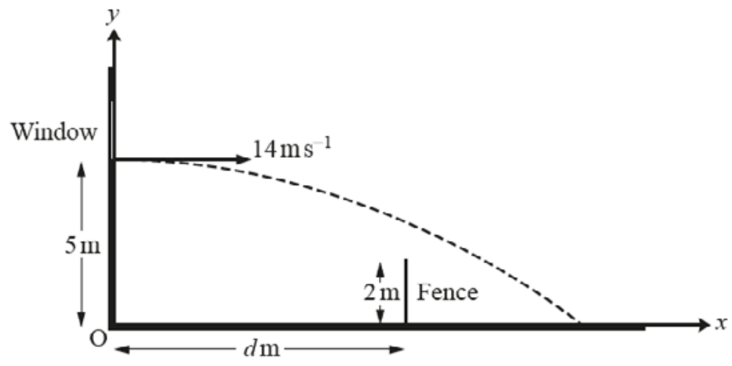


Fig. 9

(i) Find the time the pebble takes to reach the ground. **[3]**

(ii) Find the cartesian equation of the trajectory of the pebble. **[4]**

(iii) Find the range of possible values for d. **[3]**

Exemplar 1

(i) 3 marks

9(i)

$S_y = ut + \frac{1}{2}at^2$

$-5 = -4.9t^2$

$\cancel{0 = 2 - 4.9t^2 + 5}$

$t^2 = 1.02\ldots$

$t = \frac{5\sqrt{2}}{7} = 1.01$ seconds

(ii) 2 marks

9(ii)

$y = -4.9t^2$ B0 $\quad x = 14t$

$\frac{x}{14} = t$ B1

$y = -\frac{4.9x^2}{196}$ M1

$y = -0.025x^2$ A0

(iii) 3 marks

9(iii)

$-3 = -0.025x^2$

$x^2 = 120$

$x = 10.95\text{ m}$

$d \leq 10.95\text{ m from O}$

Examiner commentary

Part (i) of this question was very well done with almost all candidates realising that the initial velocity in the vertical direction was zero as the pebble is thrown horizontally. In part (ii) this candidate did not use the origin given in the question and shown on the diagram, so left out the initial position 5 m above the origin in the equation for *y*. However, their working was clear and the elimination of *t* from the equations is clearly seen so the method mark was credited. A consistent application of their model allowed a successful response in part (iii).

Exemplar 2

(iii) 2 marks

9(iii)

$x = 14 \times 1.01 = 14.14 \quad d < 14.14\text{m}$

$-3 = 0 - \cancel{\ } 4.9t^2$

$-3 = -4.9t^2$

$t^2 = 0.6122$

$t = \pm 0.78246\text{s}$ (can't have negative time)

$t = 0.78246\text{s}$

$x = 14 \times 0.78246\text{s} = 10.954\text{m}$ M1 A1

$= 11.0\text{ m (3sf)}$ A0

^

Examiner commentary

In part (iii) the simplest method was to use the equation of the trajectory found in part (ii) – often a question is structured to indicate a method to solve a problem – quite often the word "Hence" is used to link part questions together. This candidate chose not to use the equation from part (ii) and solved the problem directly from considering the vertical and horizontal directions separately. The correct value is obtained for the largest distance that the wall could be from the window. A mark was lost here as the question asked for a range of values, so an inequality was needed for full credit.

Question 10

10 Fig. 10 shows the graph of $y = (k - x)\ln x$ where k is a constant ($k > 1$).

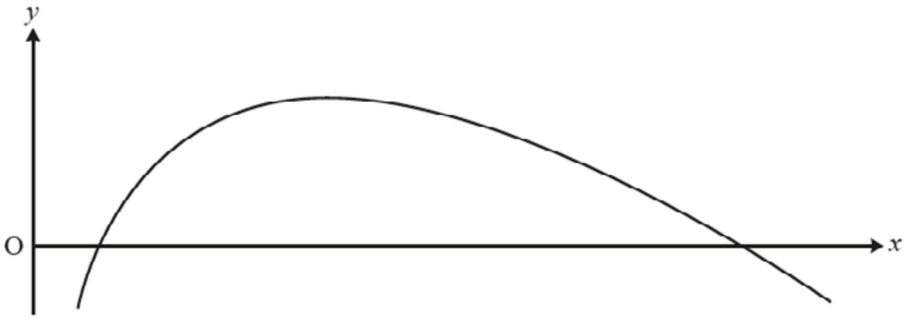


Fig. 10

Find, in terms of k, the area of the finite region between the curve and the x-axis. **[8]**

Exemplar 1 — 5 marks

10

$y = (k - x)\ln x$

$y = 0 \quad (k - x)\ln x = 0$

$\ln x = 0 \qquad k - x = 0$

$x = e^0 \qquad x = k$ M1 A1

$x = 1$

$\int_1^k (k - x)\ln x \, dx$

$u = \ln x \qquad \frac{dv}{dx} = k - x$

$\frac{du}{dx} = 1/x \qquad v = kx - \frac{x^2}{2}$ M1 A1

$\int_1^k (k-x)\ln x \, dx = \left[\ln x\left(kx - \frac{x^2}{2}\right)\right]_1^k - \int_1^k \frac{k}{x} - 1 \, dx$ M0

$= \left[\ln x\left(kx - \frac{x^2}{2}\right) - k\ln x + x\right]_1^k$

$= \cancel{\ln}\left(\ln k\left(k^2 - \frac{k^2}{2}\right) - k\ln k + k\right) - \left(\ln 1\left(k - \frac{1}{2}\right) - k\ln 1 + 1\right)$ M1

$= \ln k\left(\frac{k^2}{2}\right) - k\ln k + k - 1$

$= \ln k\left(\frac{k^2}{2} - k\right) + k - 1$ A0

Examiner commentary

Question 10 requires a structured answer from candidates and there are several steps of working needed. Some candidates realised they had to find the points of intersection with the *x*-axis and then integrate using integration by parts. It is important that candidates who are not able to find the limits do not give up on this question but find the indefinite integral even if they are unable to finish the question. In this exemplar, the limits are found successfully and a good attempt is made at the integration. There is a mix-up with the fractions, which costs three marks in total. Notice that the substitution of limits into an incorrect expression is very clear so the examiner was able to award the method mark for using the limits even though the answer was incorrect – had the candidate cut corners here, this mark might also have been lost.

Question 11 (i) and (ii)

11 Fig. 11 shows two blocks at rest, connected by a light inextensible string which passes over a smooth pulley. Block A of mass 4.7 kg rests on a smooth plane inclined at 60° to the horizontal. Block B of mass 4 kg rests on a rough plane inclined at 25° to the horizontal. On either side of the pulley, the string is parallel to a line of greatest slope of the plane. Block B is on the point of sliding up the plane.

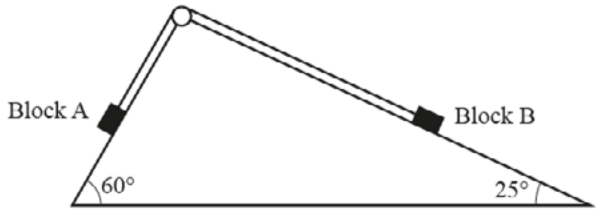


Fig. 11

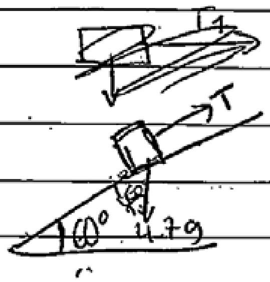
(i) Show that the tension in the string is 39.9 N correct to 3 significant figures. **[2]**

(ii) Find the coefficient of friction between the rough plane and Block B. **[5]**

Exemplar 1

(i) 2 marks

11(i) Block A



$T = 4.7g\sin 60$

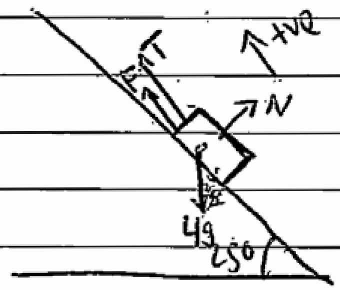
$T = 39.8891301$

$T = 39.9\text{ N}$

(ii) 3 marks

11(ii)

$F = \mu N$



finding F

$F + T = 4g \sin 25$ M1 A0

$F + 39.9 = 4g \sin 25$

$F = -23.333...$

$F = -23.3$

$N = 4g \cos 25$

$N = 35.52726525$ B1

$-23.3... = \mu\, 35.57726.$ M1

$-0.66 = \mu$ A0

Examiner commentary

This exemplar shows the advantages of drawing a clear force diagram to help in a Mechanics question. There is an error in the diagram for part (ii) – the block is on the point of moving up the plane, so the friction force should be acting in the opposite direction. However, the value for the friction force is now negative so the mistake could be overcome by interpreting the negative value – stating that the friction force is in the opposite direction to that shown on the diagram and used in the equilibrium equation. This candidate simply uses the negative value without comment and produces a negative value for the coefficient of friction, which is not possible.

Question 12 (ii) and (iii)

12 Fig. 12 shows the circle $(x-1)^2+(y+1)^2=25$, the line $4y=3x-32$ and the tangent to the circle at the point A (5, 2). D is the point of intersection of the line $4y=3x-32$ and the tangent at A.

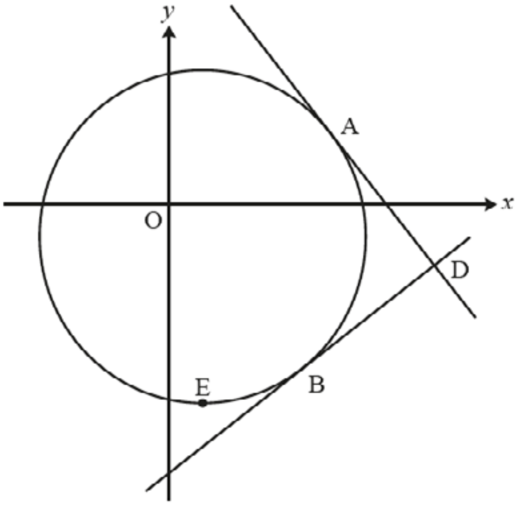


Fig. 12

(i) Write down the coordinates of C, the centre of the circle. **[1]**

(ii) (*A*) Show that the line $4y=3x-32$ is a tangent to the circle. **[4]**

(*B*) Find the coordinates of B, the point where the line $4y=3x-32$ touches the circle. **[1]**

(iii) Prove that ADBC is a square. **[3]**

(iv) The point E is the lowest point on the circle. Find the area of the sector ECB. **[5]**

Exemplar 1

(ii)(*A*) — 4 marks

$y = \frac{3}{4}x \cancel{+} -8$

$(x-1)^2 + (\frac{3}{4}x - 7)^2 = 25$

$\frac{25}{16}x^2 - \frac{\cancel{4}50}{4}x + 50 = 25$

$x^2 - 8x + 32 = 16$

$x^2 - 8x + 16 = 0$

$(x-4)^2 = 0$ ✓

so the line and the circle only meet at 1 point and the line is tangent. ✓

(ii)(*B*) 1 mark

12(ii)(*B*)	$x = 4$
	$4y = 3 \cdot 4 - 32$
	$4y = -20$
	$y = -5$
	$(4, -5)$ ✓

Examiner commentary

This exemplar demonstrates the minimum amount of working for parts (ii) (*A*) and (ii) (*B*). The repeated factor was clearly shown so the statement at the end was just sufficient for the final mark. Ideally, the comment should have stated that there is a repeated root so that the line is a tangent to the circle. Notice part (ii) (*B*) is only worth one mark as most of the work has already been done in part (ii) (*A*).

Exemplar 2

(iii) 3 marks

12(iii)	$A(5,2),\ B(4,-5),\ C(1,-1)$	~~Math~~ cont.
	$x^2-2x+1+y^2+2y+1=25$	$\vec{AD}=\binom{3}{-4}$
	$x^2-2x+y^2+2y-23=0$	$\vec{CB}=\binom{3}{-4}$
	$2x-2+2y\frac{dy}{dx}+2\frac{dy}{dx}=0$	$\vec{BD}=\binom{4}{3}$
	at $x=5, y=2$	$\vec{CA}=\binom{4}{3}$
	$10-2+4\frac{dy}{dx}+2\frac{dy}{dx}=0$	
	$8+6\frac{dy}{dx}=0$	$a^2+b^2=c^2$
	$\frac{dy}{dx}=\frac{-8}{6}=-\frac{4}{3}$	$\lvert\vec{AB}\rvert=\lvert\vec{CB}\rvert=\lvert\vec{BD}\rvert=\lvert\vec{CA}\rvert$
	$y-y_1=m(x-x_1)$	$=5$ so all same length
	$y-2=-\frac{4}{3}(x-5)$	$\vec{AD}=\vec{CB}$ and $\vec{BD}=\vec{CA}$
	$3y-6=-4x+20$	$\therefore$ 2 pairs of parallel
	$3y=-4x+26$	sides
	$y=\frac{3}{4}x-8$	gradient of $\vec{AD}=\frac{-4}{3}$
	$\frac{9}{4}x-24=-4x+26$	gradient of $\vec{BD}=\frac{3}{4}$
	$\frac{9}{4}x+4x=50$	~~gradient of AD~~
	$\frac{25}{4}x=50$	$\frac{-4}{3}$ is -ve reciprocal of $\frac{3}{4}$
	$x=8$	$\therefore$ perpendicular
	$y=\frac{3}{4}x-8$	
	$y=-2$	So must be a square ✓
	$D(8,-2)$	

Examiner commentary

This candidate correctly uses implicit differentiation to find the gradient of the tangent at A and so has a complete proof. This would rarely have been seen in a modular examination when the coordinate geometry of circles was part of the AS specification and implicit differentiation A2.

Exemplar 3

(iii) 2 marks

12(iii)

$A = (5, 2)$

$B = (4, -5)$

$C = (1, -1)$

$D = (8, -2)$

~~$AB = \sqrt{1 + 7^2} = \sqrt{50}$~~

~~$BC = \sqrt{3^2 + 4^2} =$~~

$AD = \sqrt{3^2 + 4^2} = 5$

$DB = \sqrt{4^2 + 3^2} = 5$

$BC = \sqrt{4^2 + 3^2} = 5$

$CA = \sqrt{4^2 + 3^2} = 5$

all sides are equal, therefore ADBC is a square

B1

^

$y = -4/3\,x + c$

$2 = (-4/3 \times 5) + c$

$2 = -20/3 + c$

$c = 26/3$

$y = -4/3\,x + 26/3$

$y = 3/4\,x - 8$

$3/4\,x - 8 = -4/3\,x + 26/3$

$9/4\,x - 24 = -4x + 26$

$9/4\,x = -4x + 50$

$25/4\,x = 50$

$x = 8$

$4y = 24 - 32$

$y = -2$

$D = (8, -2)$

Examiner commentary

It might appear at first glance that there is a perfect proof here but it is in fact a circular argument. There is no justification for choosing the gradient of $-\frac{4}{3}$ seen here so the examiner must assume that it comes from the gradient of the given line, assuming them to be perpendicular. The candidate then finds the coordinates of D and argues that four equal sides proves that ABCD is a square. There is no mention in the argument of at least one right angle. A much better way of finding the gradient of the tangent is to find the gradient of AC and make sure the tangent is perpendicular to this. In fact, it is not necessary to find the equation of AC at all and the proof can be completed without finding the coordinates of D.

Exemplar 4

(iii) 2 marks

12(iii)

$\overline{CA} = \overline{CB}$ radius of the circle B1

BD and AD are tangents and have the same length

B1

tangent meets radius at 90° tangents meet at 90°

∠ADB is 90° therefore 4 all angles are 90°

Then ADBC is a square and equal sides

Examiner commentary

In this exemplar, two of the three facts that prove ABCD is square are neatly given. The statement that the tangents meet at 90^0 needs to be proved and not simply stated so the proof is incomplete.

Question 13 (ii), (iii) and (v)

13 The function $f(x)$ is defined by $f(x) = \sqrt[3]{27 - 8x^3}$. Jenny uses her scientific calculator to create a table of values for $f(x)$ and $f'(x)$.

x	$f(x)$	$f'(x)$
0	3	0
0.25	2.9954	–0.056
0.5	2.9625	–0.228
0.75	2.8694	–0.547
1	2.6684	–1.124
1.25	2.2490	–1.977
1.5	0	ERROR

(i) Use calculus to find an expression for $f'(x)$ and hence explain why the calculator gives an error for $f'(1.5)$. **[3]**

(ii) Find the first three terms of the binomial expansion of $f(x)$. **[3]**

(iii) Jenny integrates the first three terms of the binomial expansion of $f(x)$ to estimate the value of $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. Explain why Jenny's method is valid in this case. (You do not need to evaluate Jenny's approximation.) **[2]**

(iv) Use the trapezium rule with 4 strips to obtain an estimate for $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. **[3]**

The calculator gives 2.921 174 38 for $\int_0^1 \sqrt[3]{27 - 8x^3}\,dx$. The graph of $y = f(x)$ is shown in Fig. 13.

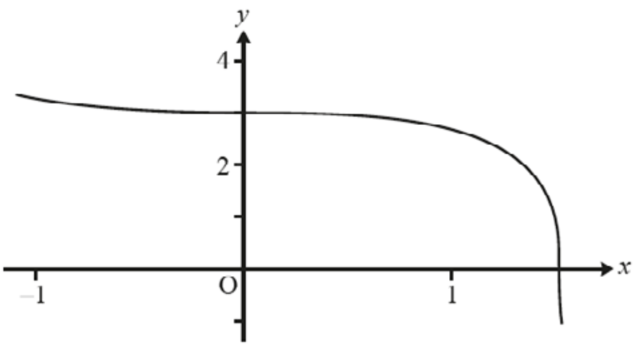


Fig. 13

(v) Explain why the trapezium rule gives an underestimate. **[1]**

Exemplar 1

(ii) 1 mark

13(ii)

$(27-8x^3)^{1/3}$

B0

$3\left(1-\frac{8}{27}x^3\right)^{1/3}$

$1+\left(\frac{1}{3}\right)\left(\frac{-8}{27}x^3\right)+\frac{1/3(1/3-1)}{2}\left(-\frac{8}{27}x^3\right)^2$ M1

$1 - \frac{8}{81}x^3 - \frac{64}{6561}x^6$ A0

Examiner commentary

Part (i) was well answered with most candidates accurately using the chain rule for differentiation and realising this put a zero in the denominator at $x = 1.5$. Part (ii) caused more difficulty and this exemplar shows the common mistake seen dealing correctly with the 27.

Exemplar 2

(ii) 1 mark

13(ii)

$(27-8x^3)^{1/3}$

$27^{1/3}\left(1-\frac{8}{27}x^3\right)^{1/3}$

M0

$3\left(1-8/27\,x^3\right)^{1/3} \quad x \quad 1 - \frac{8}{81}x^3 \mp \frac{8}{243}x^6 \ldots$

B1

Examiner commentary

This candidate has probably got the right idea about the binomial expansion with a mistake in the arithmetic. However, there isn't enough working seen to be sure so the method mark is lost as well as the final accuracy mark because the examiner cannot assume a correct method from an incorrect answer.

Exemplar 3

(iii) 2 marks

13(iii)

$27 - 8x^3$

$-8x^3 < -27$

$x^3 = \frac{-27}{-8}$

$x = \sqrt[3]{\frac{27}{8}} = \frac{3}{2}$

the expension $|x| < \frac{3}{2}$
is valid for $|x| < \frac{3}{2}$ And as she is working with values $0 \leq x \leq 1$ ~~sidenars saying...~~ the intergration method is valid.

Examiner commentary

Many candidates were very vague in their explanation here and received no marks. There is a clue in the mark allocation that more than a simple sentence is needed – this exemplar is a good example of a candidate who realises all that is involved – a calculation of the interval for which the binomial expansion is valid and a comment that the limits of the integral lie well within that interval.

Exemplar 1

(v) 1 mark

13(v)

The curve is concave downwards so all of the trapeziums lie slightly underneath the curve, thus causing an underestimate.

Examiner commentary

Most candidates could use the trapezium rule and the given values to obtain the correct answer in part (iv). This exemplar is a good example of what was required in part (v). Some candidates lost the mark here as they simply stated that the curve is concave downwards, which was not considered to be sufficient explanation.

Question 14 (i), (ii), (iii), (iv), (vi) and (vii)

14 The velocity of a car, $v\,\mathrm{m\,s^{-1}}$ at time t seconds, is being modelled. Initially the car has velocity $5\,\mathrm{m\,s^{-1}}$ and it accelerates to $11.4\,\mathrm{m\,s^{-1}}$ in 4 seconds.

In model A, the acceleration is assumed to be uniform.

(i) Find an expression for the velocity of the car at time t using this model. **[3]**

(ii) Explain why this model is not appropriate in the long term. **[1]**

Model A is refined so that the velocity remains constant once the car reaches $17.8\,\mathrm{m\,s^{-1}}$.

(iii) Sketch a velocity-time graph for the motion of the car, making clear the time at which the acceleration changes. **[3]**

(iv) Calculate the displacement of the car in the first 20 seconds according to this refined model. **[3]**

In model B, the velocity of the car is given by

$$v = \begin{cases} 5 + 0.6t^2 - 0.05t^3 & \text{for } 0 \leqslant t \leqslant 8, \\ 17.8 & \text{for } 8 < t \leqslant 20. \end{cases}$$

(v) Show that this model gives an appropriate value for v when $t = 4$. **[1]**

(vi) Explain why the value of the acceleration immediately before the velocity becomes constant is likely to mean that model B is a better model than model A. **[3]**

(vii) Show that model B gives the same value as model A for the displacement at time 20 s. **[3]**

Exemplar 1

(i) 3 marks

14(i)	$u=5$
	$a=?$
	$v=11.4$ Use suvat
	$t=4$
	$v=u+at$
	$11.4=5+4a$
	$\frac{11.4-5}{4}=a=1.6$
	$v=\cancel{0}\ 5+1.6t$

Examiner commentary

Most candidates had a good answer in part (i). This exemplar is an example of good practice with the relevant values for the *suvat* equations clearly stated. Notice the two parts of the solution work over different time frames – using 0 to 4 s for the first part to find the acceleration, and then using the variable *t* and the calculated value for *a* to find the expression for velocity.

Exemplar 2

(ii) 0 marks

14(ii) The car probably wouldn't continue accelerating constantly

^

Examiner commentary

These exemplars give an idea of the spectrum of detail that candidates gave. This first exemplar is not enough; a practical reason why the model prediction is invalid is needed.

Exemplar 3

(ii) 1 mark

14(ii) The car wouldn't be accelerating constantly, it would need to slow down now and then ~~so~~ so it won't go too fast.

B1 BOD

Examiner commentary

This exemplar is sufficient for the mark.

Exemplar 4

(ii) 1 mark

14(ii) The car cannot constantly accelerate as it would break the laws of physics. Nothing can travel faster than the speed of light.

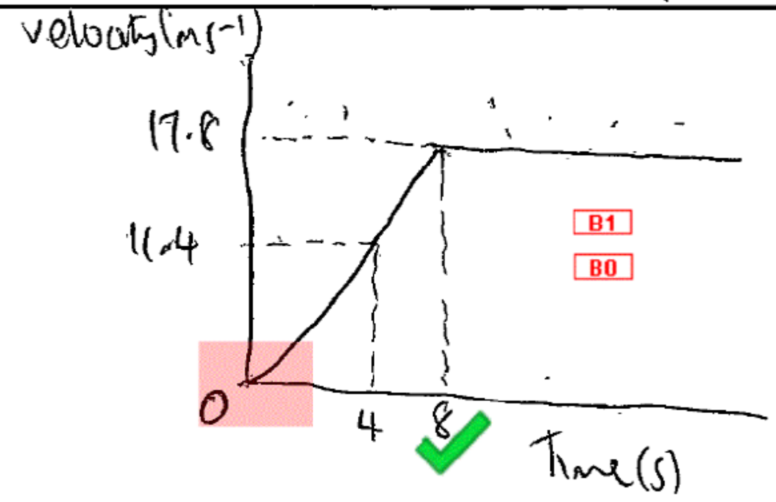
Examiner commentary

This third exemplar gives an understanding of the model and of the laws of physics beyond what is expected in A Level Maths.

Exemplar 5

(iii) 2 marks

14(iii)



$1.6t = 17.8 - 5$

$t = 8$ B1

(iv) 2 marks

14(iv)

$\frac{1}{2}(17.8)(8) + 12(17.8)$ M1 A0

$\Rightarrow 284.8$ m A1 FT

Examiner commentary

This exemplar shows a common error – beginning the graph at the origin, neglecting the initial velocity of the car. This lost one mark in part (iii), but the other two marks were both available. In part (iv) the candidate uses the area under their graph so still gets the method marking for finding the area of the triangle and the follow through mark for adding 213.6 to their answer.

Exemplar 6

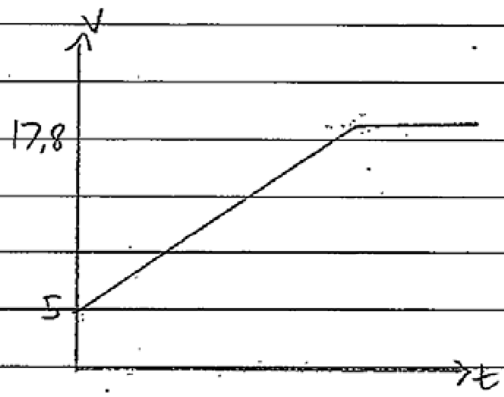
(iii) NR

14(iii)

BP

(iv) 0 marks

14(iv)



$v = u + at$

$17.8 = 5 + 1.6t$

^

Examiner commentary

This exemplar shows the importance of using the Printed Answer Booklet properly. The section for 14 (iii) is blank so the candidate is credited NR for this part. The graph does appear in part (iv) but the examiner is not allowed to award the marks for part (iii) when the answer is in part (iv) unless the candidate indicates that's where it is. The examiner must assume that the candidate did not realise that the work they did in part (iv) was relevant to part (iii).

Exemplar 7

(vi) 3 marks

14(vi)

$a = 1.2t - 0.15t^2$

$t = 7.9 \quad a = 0.1185 \text{ ms}^{-2}$

$t = 8 \quad a = 0$

It's a better model as it shows the ~~de~~ acceleration decreasing and approaching 0 the constant velocity. In model A there is a n instant between the uniform acceleration ~~and~~ of 1.6ms^{-2} and 0ms^{-2}. In reality the change would be more gradual which makes model B better. ✓

Examiner commentary

This exemplar is a very good response to the question. The question clearly states that the acceleration is key to this answer and that an explanation is required. Some candidates were very vague here with no calculus seen – there is a clue in the mark allocation that quite a bit of work is required to get full marks.

Exemplar 8

(vii) 3 marks

14(vii)

$0 \le t \le 8$

$\int_0^8 v \, dt = \left[8t + 0.2t^3 - 0.0128t^4\right]_0^8$ M1

$= 91.2\text{m}$ A1

$91.2 + 213.6 = 304.8\text{m}$ A1

area under v-t graph

Examiner commentary

This exemplar demonstrates the smart use of the calculator functions in Mechanics. The point being tested is here is the understanding that the definite integral with the correct limits gives the displacement of the car. Here the definite integral is clearly shown together with the right answer so is the minimum working required in this type of 'Show that …' question. If the question had included the statement "**In this question you must show detailed reasoning**" at the beginning then a clear substitution of the limits would be required (as in Practice Paper Set 1 H640/01 Q2).

The small print

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Qualification and notional component raw mark grade boundaries June 2018 series

The following qualifications are linear and therefore do not use UMS. For an explanation of how the new linear qualifications work, check out our blog: www.ocr.org.uk/blog/view/how-linear-qualifications-and-grade-boundaries-work

New AS Levels

AS GCE Mathematics B (MEI)										
				Max Mark	a	b	c	d	e	u
H630	01	Pure Mathematics and Mechanics	Raw	70	44	38	33	28	23	0
H630	02	Pure Mathematics and Statistics	Raw	70	50	45	39	33	28	0
			Overall	140	94	83	72	61	51	0

AS GCE Further Mathematics B (MEI) (H635)										
				Max Mark	a	b	c	d	e	u
Y410	01	Core Pure	Raw	60	46	41	36	32	28	0
Y411	01	Mechanics a	Raw	60	37	32	27	22	18	0
Y412	01	Statistics a	Raw	60	42	38	34	30	26	0
Y413	01	Modelling with Algorithms	Raw	60	37	33	29	25	22	0
Y414	01	Numerical Methods	Raw	60	35	29	24	19	14	0
Y415	01	Mechanics b	Raw		No entry in June 2018					
Y416	01	Statistics b	Raw	60	43	38	33	28	24	0
H635		*Option Y410+Y411+Y412*	Overall	180	125	111	98	85	72	0
H635		*Option Y410+Y411+Y413*	Overall	180	120	107	94	81	68	0
H635		*Option Y410+Y411+Y414*	Overall	180	118	103	88	74	60	0
H635		*Option Y410+Y412+Y413*	Overall	180	125	112	100	88	76	0
H635		*Option Y410+Y412+Y414*	Overall	180	123	109	95	81	68	0
H635		*Option Y410+Y412+Y416*	Overall	180	131	117	104	91	78	0
H635		*Option Y410+Y413+Y414*	Overall	180	118	104	90	77	64	0

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New A Levels

A Level Mathematics B (MEI)												
				Max Mark	a*	a	b	c	d	e	u
H640	01	Pure Mathematics and Mechanics	Raw	100	81	74	67	59	52	45	0
H640	02	Pure Mathematics and Statistics	Raw	100	75	68	61	54	47	40	0
H640	03	Pure Mathematics and Comprehension	Raw	75	62	55	48	42	36	30	0
			Overall	275	218	197	176	155	135	115	0