SPECIMEN MATERIAL

Please write clearly, in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname _____

Forename(s) _____

Candidate signature _____

A-level
FURTHER MATHEMATICS

Paper 2

Exam Date | Morning | Time allowed: 2 hours

Materials

For this paper you must have:
- The AQA booklet of formulae and statistical tables.
- You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should be used for drawing.
- Answer **all** questions.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 100.

Advice

Unless stated otherwise, you may quote formulae, without proof, from the booklet.
You do not necessarily need to use all the space provided.

Answer **all** questions in the spaces provided.

1 Given that $z_1 = 4e^{i\frac{\pi}{3}}$ and $z_2 = 2e^{i\frac{\pi}{4}}$

state the value of $\arg\left(\frac{z_1}{z_2}\right)$

Circle your answer.

[1 mark]

$\frac{\pi}{12}$ $\frac{4}{3}$ $\frac{7\pi}{12}$ 2

2 Given that z is a complex number and that z^* is the complex conjugate of z

prove that $zz^* - |z|^2 = 0$

[3 marks]

3 The transformation T is defined by the matrix **M**. The transformation S is defined by the matrix $\mathbf{M}^{-1}$. Given that the point (x, y) is invariant under transformation T, prove that (x, y) is also an invariant point under transformation S.

[3 marks]

4 Solve the equation $z^3 = \mathrm{i}$, giving your answers in the form $\mathrm{e}^{\mathrm{i}\theta}$, where $-\pi < \theta \leq \pi$

[4 marks]

5 Find the smallest value θ of for which

$$(\cos\theta + \mathrm{i}\sin\theta)^5 = \frac{1}{\sqrt{2}}(1 - \mathrm{i}) \quad \{\theta \in \mathbb{R} : \theta > 0\}$$

[4 marks]

6 Prove that $8^n - 7n + 6$ is divisible by 7 for all integers $n \geq 0$

[5 marks]

7 A small, hollow, plastic ball, of mass m kg is at rest at a point O on a polished horizontal surface. The ball is attached to two identical springs. The other ends of the springs are attached to the points *P* and *Q* which are 1.8 metres apart on a straight line through O.

The ball is struck so that it moves away from O, towards *P* with a speed of 0.75 m s^{-1}.

As the ball moves, its displacement from O is x metres at time t seconds after the motion starts.

The force that each of the springs applies to the ball is $12.5mx$ newtons towards O.

The ball is to be modelled as a particle. The surface is assumed to be smooth and it is assumed that the forces applied to the ball by the springs are the only horizontal forces acting on the ball.

7 (a) Find the minimum distance of the ball from *P*, in the subsequent motion.

[5 marks]

7 (b) In practice the minimum distance predicted by the model is incorrect.

Is the minimum distance predicted by the model likely to be too big or too small?

Explain your answer with reference to the model.

[2 marks]

8 Given that $I_n = \int_0^{\frac{\pi}{2}} \sin^n x \, \mathrm{d}x \qquad n \geq 0$

show that $n\, I_n = (n-1) I_{n-2} \qquad n \geq 2$

[5 marks]

9 A student claims:

"Given any two non-zero square matrices, **A** and **B**, then $(\mathbf{AB})^{-1} = \mathbf{B}^{-1}\mathbf{A}^{-1}$"

9 (a) Explain why the student's claim is incorrect giving a counter example.

[2 marks]

9 (b) Refine the student's claim to make it fully correct.

[1 mark]

9 (c) Prove that your answer to part **(b)** is correct.

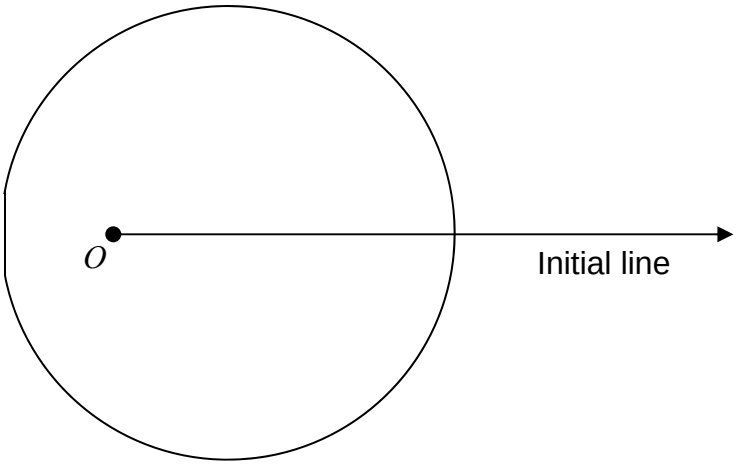
[3 marks]

10 Evaluate the improper integral $\int_0^\infty \frac{4x - 30}{(x^2 + 5)(3x + 2)} \mathrm{d}x$, showing the limiting process used.

Give your answer as a single term.

[8 marks]

11 The diagram shows a sketch of a curve *C*, the pole O and the initial line.



The polar equation of *C* is $r = 4 + 2\cos\theta$, $-\pi \leq \theta \leq \pi$

11 (a) Show that the area of the region bounded by the curve *C* is 18π

[4 marks]

11 (b) Points A and B lie on the curve C such that $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$ and AOB is an equilateral triangle.

Find the polar equation of the line segment AB

[4 marks]

12 $\mathbf{M} = \begin{bmatrix} -1 & 2 & -1 \\ 2 & 2 & -2 \\ -1 & -2 & -1 \end{bmatrix}$

12 (a) Given that 4 is an eigenvalue of **M**, find a corresponding eigenvector.

[3 marks]

12 (b) Given that **MU** = **UD**, where **D** is a diagonal matrix, find possible matrices for **D** and **U**.

[8 marks]

13 **S** is a singular matrix such that

$$\det \mathbf{S} = \begin{vmatrix} a & a & x \\ x-b & a-b & x+1 \\ x^2 & a^2 & ax \end{vmatrix}$$

Express the possible values of x in terms of a and b.

[7 marks]

14 Given that the vectors **a** and **b** are perpendicular, prove that

$|(\mathbf{a} + 5\mathbf{b}) \times (\mathbf{a} - 4\mathbf{b})| = k|\mathbf{a}||\mathbf{b}|$, where k is an integer to be found.

Explicitly state any properties of the vector product that you use within your proof.

[9 marks]

15 (a) Show that $(1-\frac{1}{4}e^{2i\theta})(1-\frac{1}{4}e^{-2i\theta}) = \frac{1}{16}(17-8\cos 2\theta)$

[3 marks]

15 (b) Given that the series $e^{2i\theta} + \frac{1}{4}e^{4i\theta} + \frac{1}{16}e^{6i\theta} + \frac{1}{64}e^{8i\theta} + ...$ has a sum to infinity, express this sum to infinity in terms of $e^{2i\theta}$

[2 marks]

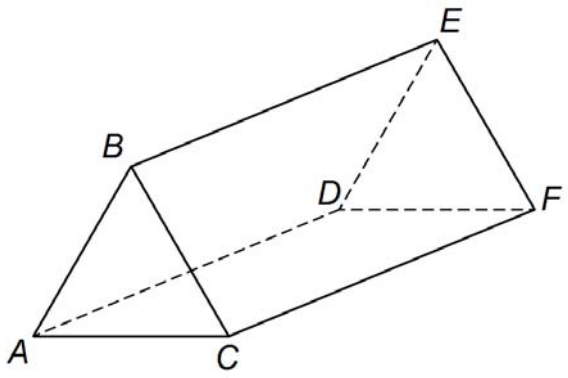
15 (c) Hence show that $\sum_{n=1}^{\infty} \frac{1}{4^{n-1}} \cos 2n\theta = \frac{16\cos 2\theta - 4}{17 - 8\cos 2\theta}$

[4 marks]

15 (d) Deduce a similar expression for $\sum_{n=1}^{\infty} \frac{1}{4^{n-1}} \sin 2n\theta$

[1 mark]

16 A designer is using a computer aided design system to design part of a building. He models part of a roof as a triangular prism *ABCDEF* with parallel triangular ends *ABC* and *DEF*, and a rectangular base *ACFD*. He uses the metre as the unit of length.



The coordinates of *B*, *C* and *D* are (3, 1, 11), (9, 3, 4) and (–4, 12, 4) respectively.

He uses the equation $x - 3y = 0$ for the plane *ABC*.

He uses $\left[\mathbf{r} - \begin{pmatrix} -4 \\ 12 \\ 4 \end{pmatrix}\right] \times \begin{pmatrix} 4 \\ -12 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$ for the equation of the line *AD*.

Find the volume of the space enclosed inside this section of the roof.

[9 marks]

END OF QUESTIONS